A Statistical Method of Accelerated Life Testing Based on Fuzzy Theory

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Abstract—**Failure data in accelerated life test are imprecise due to kind of subjectivity. This makes the assessment of testing lack of accuracy. A statistical model of accelerated life testing based on fuzzy theory is proposed in this paper. The model aims to describe the subjectivity of failure data and give a fuzzy interval assessment of lifetime and reliability comparing to the traditional point estimation. Firstly, triangular membership function is chosen to describe Type II censored data and their α-level cut sets are settled under acceptable level. Then, the statistical modelling for ALT data is established based on fuzzy theory by integrating maximum likelihood estimation to determine the membership functions of parameters. And using Particle Swarm Optimization algorithm calculates their fuzzy evaluation values. Finally, the proposed method is verified by the simulation study.**

Keywords- accelerated life testing; fuzzy theory; Particle Swarm Optimization; Maximum Likelihood Estimation; reliability.

ACRONYM

ALT	Accelerated Life Testing
PSO	Particle Swarm Optimization
EA	Evolutionary Algorithm
MLE	Maximum Likelihood Estimation
CSALT	constant-stress ALT
SSALT	step-stress ALT
PSALT	progressive-stress ALT

NOTATION

$F(t)$	Cumulative distribution function
λ	Failure rate function
θ	Mean life
$R(t)$	Reliable distribution function
S_i	Stress level ith
T_i	Total testing time under stress S_i
r_i	Number of samples fail under stress S_i
k	Number of stress level
n_i	Number of samples under stress S_i

I. INTRODUCTION

Accelerated life testing is widely used for the life and reliability evaluation of high-reliability and long-life products as traditional life testing is hard to efficiently accomplish the tasks. Accelerated life testing(ALT) contains three kinds of testing: constant-stress ALT(CSALT), step-stress ALT(SSALT) and progressive-stress ALT(PSALT). Current statistical methods of those ALT can be divided into Bayesian statistical analysis and classical statistical methods which contains maximum likelihood estimation(MLE), least square estimation , linear estimations based on order statistics and so on. These methods require failure data analysis.

In ALT, periodic inspection is always conducted to monitor the product performance, obtaining the failure time data. However, the failure time cannot be accurate since it may happen at the adjacent inspection interval. As a result the failure time of samples is not precise because of being recorded subjectively. Thus, there exists big uncertainty in failure time data if the interval is quite long. When the ALT data is extrapolated to normal conditions, the uncertainty will also transfer and affect the accuracy of evaluation results, lacking of credibility.

Fuzzy theory is one of the important methods dealing with uncertainty. It has been widely studied by international researchers. Viertl [1-2] considers reliability of the system based on Bayesian theory when the lifetime data are fuzzy. Hong-Zhong Huang studies the similar case[3] and also proposes a reliability model on competitive failure processes under fuzzy degradation data[4]. Jamkhaneh[5-6] particularly describes three kinds of lifetime distribution: Binomial distribution, Exponential distribution and Weibull distribution when the parameters of the distributions are fuzzy. The reliability of the system with fuzzy parameters is investigated. Lin[7] provides a membership function of the characteristic of a repairable system with fuzzy parameters of Exponential lifetime distribution.

In this paper, a statistical method for constant-stress ALT with Type II censored data based on fuzzy theory is proposed The lifetime distribution of products follows exponential distribution. Maximum likelihood estimation is used for data statistics.

II. Accelerated Life Testing anf Fuzzy Theory

A. The statistical model of accelerated life testing

A statistical model of some product with exponential lifetime distribution is presented in this paper. The statistical inference of ALT bases on two assumptions. Firstly, the lifetime distribution of the product will always be exponential distribution under the normal stress level and accelerated stress levels. It can be expressed as

$$\begin{aligned} &F_i(t) = 1 - \exp(-t/\theta_i) \\ &t > 0; i = 0,1,\dots,k \end{aligned} \tag{1}$$

In the Eq.(1), θ_i represents the mean life of the product under the stress level S_i. Secondly, the relationship between the mean life θ_i and accelerated stress level S_i ,also known as the accelerated model, is

$$\ln\theta_i = a + b\varphi(S_i) \tag{2}$$

Where a and b are parameters to be estimated. $\varphi(Si)$ is a function of stress S_i depending on the accelerated model.

In this paper, we choose the CSALT as an example owing to its universality in engineering. In CSALT, assuming there are n samples working at constant but different stress levels. Presuming an ALT has k accelerated stress levels, denoted as $S_1<S_2<\ldots<S_k$, normal stress level is S_0. At each accelerated stress level S_i, n_i specimens are run to failure until r_i specimens failed. The failure time data are called Type II censored data. Mark failure times of jth sample under its accelerated stress as t_{ij}, i=1,2,…,k, j=1,2, …, n_i. The total testing time of accelerated stress level S_i is

$$T_i = \sum_{j=1}^{r_i} t_{ij} + (n_i - r_i)t_{ir_i} \tag{3}$$

Then the mean time θ_i of samples under stress level S_i can be calculated as

$$\theta_i = T_i / r_i \tag{4}$$

And the likelihood function under stress level S_i is

$$f = \lambda_i^{r_i}\exp(-\lambda_i T_i) \tag{5}$$

Then the likelihood function of the total accelerated life testing is

$$L = \left(\prod_{i=1}^{k}\lambda_i^{r_i}\right)\exp\left(-\sum_{i=1}^{k}\lambda_i T_i\right) \tag{6}$$

Combine Eq.(2) and Eq.(7), we can get

$$\ln L(a,b) = -a\sum_{i=1}^{k} r_i - b\sum_{i=1}^{k}\varphi(S_i)r_i - \sum_{i=1}^{k}T_i e^{-(a+b\varphi(S_i))} \tag{7}$$

After some deductions of Eq.(7), Eq.(8)is obtained as fellows.

$$\begin{aligned} &a = -\ln\left(\sum_{i=1}^{k} r_i\right) + \ln\left[\sum_{i=1}^{k} T_i\exp(-b\varphi(S_i))\right] \\ &\left(\sum_{i=1}^{k} r_i\right)\left[\sum_{i=1}^{k} T_i\varphi(S_i)\exp(-b\varphi(S_i))\right] = \left(\sum_{i=1}^{k}\varphi(S_i)r_i\right)\left[\sum_{i=1}^{k} T_i\exp(-b\varphi(S_i))\right] \end{aligned} \tag{8}$$

Traditionally, Newton Raphson iteration method is used to solve Eq.(8) and obtain the result of parameter a and b .Put a b and S_0 into Eq.(2). The mean life θ_0 under normal stress will be achieved.

B. Fuzzy theory

Fuzzy theory was proposed by Lotfi A. Zadeh in 1965s[8] and soon becomes one of the most important theories dealing with uncertainty. In this paper we just introduce some necessary concepts we will use.

A fuzzy number $\tilde{x}$ is characterized by a so-called characterizing function $\mu(.)$ which denotes the degree of membership of element $\tilde{x}$ of the universe X.

$$\begin{aligned} &\tilde{x} : X \to 0:1 \\ &\mu_{\tilde{x}}(X) \in [0,1] \end{aligned} \tag{9}$$

An α-cut of $\tilde{x}$, written as $\tilde{x}_\alpha$, is defined as

$$\begin{aligned} &\tilde{x}_\alpha = \{x \mid \mu_{\tilde{x}}(X) \ge \alpha\} \\ &0 \le \alpha \le 1 \end{aligned} \tag{10}$$

To a certain α-cut of $\tilde{x}$, the fuzzy number $\tilde{x}$ can be written as a finite closed interval as Eq.(11).

$$\tilde{x}_\alpha \in \left[\tilde{x}_\alpha^L, \tilde{x}_\alpha^U\right] \tag{11}$$

III. The Optimizaiton Model of ALT Based on Fuzzy Theory

A. Statistical model of ALT based on fuzzy theory

As discussed in the first part, Type II censored failure data in ALT are suspected because of subjectivity. The failure of samples happens at the adjacent inspection interval but the exact timing cannot be detected. Therefore, based on fuzzy theory, symmetric triangular membership function is chosen to describe the degree of recorded failure time of the exact failure timing. In this paper, failure time will be written together with its membership function as

$$\tilde{t}_{ij}=\left(m_{t_{ij}},g_{t_{ij}}\right)\left(i=1,2,...k;j=1,2,...r_i\right)$$
$$\mu_{\tilde{t}_{ij}}\left(t_{ij}\right)=\begin{cases}1-\dfrac{\left|t_{ij}-m_{t_{ij}}\right|}{g_{t_{ij}}} & m_{t_{ij}}-g_{t_{ij}}\le t_{ij}<m_{t_{ij}}+g_{t_{ij}}\\ 0 & \text{else}\end{cases} \tag{12}$$

Where $m_{t_{ij}}$ represents the center value of $\tilde{t}_{ij}$ and $g_{t_{ij}}$ represents the spread width. The value of $g_{t_{ij}}$ is greater than or equal to zero and in this paper it cannot bigger than the inspection interval. The inspection interval varies under different stress levels.

Hence, according to the fuzzy algorithm, the total testing time of accelerated stress level S_i will be changed into

$$\begin{aligned}\tilde{T}_i&=\sum_{j=1}^{r_i}\tilde{t}_{ij}=\left(\sum_{j=1}^{r_i}m_{t_{ij}},\sum_{j=1}^{r_i}g_{t_{ij}}\right)+\left(\left(n_i-r_i\right)m_{t_{ir_i}},\left|\left(n_i-r_i\right)\right|g_{t_{ir_i}}\right)\\&=\left(\sum_{j=1}^{r_i}m_{t_{ij}}+\left(n_i-r_i\right)m_{t_{ir_i}},\sum_{j=1}^{r_i}g_{t_{ij}}+\left(n_i-r_i\right)g_{t_{ir_i}}\right)\\ i&=1,2,...k\end{aligned} \tag{13}$$

$\tilde{T}_i$ is also a symmetric triangular fuzzy number on the basis of its form . Given a α-cut level, $\tilde{T}_i$ can also be written as

$$\left(\tilde{T}_i\right)_\alpha\in\left[\left(\tilde{T}_i\right)_\alpha^L,\left(\tilde{T}_i\right)_\alpha^U\right] \tag{14}$$

As a result, the Eq.(8) changes into

$$\begin{aligned}&\tilde{a}_\alpha=-\ln\left(\sum_{i=1}^{k}r_i\right)+\ln\left[\sum_{i=1}^{k}\left(\tilde{T}_i\right)_\alpha\exp\left(-\tilde{b}_\alpha\Phi_i\right)\right]\\&\left(\sum_{i=1}^{k}r_i\right)\left[\sum_{i=1}^{k}\left(\tilde{T}_i\right)_\alpha\Phi_i\exp\left(-\tilde{b}_\alpha\Phi_i\right)\right]=\left(\sum_{i=1}^{k}\Phi_i r_i\right)\left[\sum_{i=1}^{k}\left(\tilde{T}_i\right)_\alpha\exp\left(-\tilde{b}_\alpha\Phi_i\right)\right]\end{aligned} \tag{15}$$

According to the Eq.(15) and fuzzy algorithm, $\tilde{a}$ and $\tilde{b}$ are no longer symmetric triangular fuzzy numbers. The form of their membership function is hard to describe. So Particle Swarm Optimization(PSO) is chosen to ascertain the range of parameter *a* and *b.*

B. Model optimization of ALT based on PSO

Particle Swarm Optimization(PSO) is kind of Evolutionary Algorithm. It derives from birds foraging behaviors. Its core concept is that using times of iteration to find the optimal value when given a random initial value. Details will be discussed combing with the figure as below. Fig.1 shows how to optimize the ALT statistical model based on PSO.

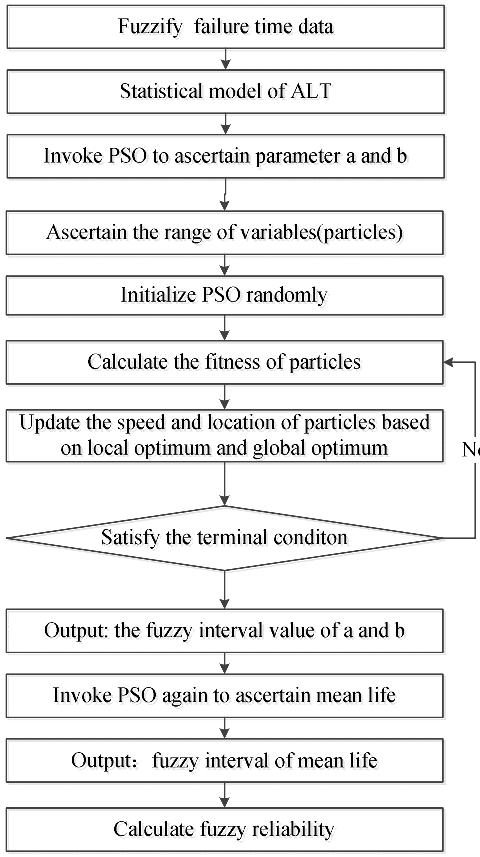

Figure 1. Optimize statistical model of ALT based on PSO

The final results of $\tilde{a}$ and $\tilde{b}$ vary When $\tilde{T}_i$ changes within its fuzzy interval according to Eq.(15) . By using PSO, the value of $\tilde{a}$ and $\tilde{b}$ is calculated as follows.

$$\begin{aligned}\tilde{a}_\alpha^L&=\min\left\{G\left(\tilde{a},\tilde{b}\right)=0:\tilde{T}\in C_\alpha\left(\tilde{T}\right)\right\}\\ \tilde{b}_\alpha^L&=\min\left\{G\left(\tilde{a},\tilde{b}\right)=0:\tilde{T}\in C_\alpha\left(\tilde{T}\right)\right\}\end{aligned} \tag{16}$$

and

$$\begin{aligned}\tilde{a}_\alpha^U&=\max\left\{G\left(\tilde{a},\tilde{b}\right)=0:\tilde{T}\in C_\alpha\left(\tilde{T}\right)\right\}\\ \tilde{b}_\alpha^U&=\max\left\{G\left(\tilde{a},\tilde{b}\right)=0:\tilde{T}\in C_\alpha\left(\tilde{T}\right)\right\}\end{aligned} \tag{17}$$

Where

$$\begin{aligned}&C_\alpha\left(\tilde{T}\right)=\left[\left(\tilde{T}_i\right)_\alpha^L,\left(\tilde{T}_i\right)_\alpha^U\right]\\&\left\{G\left(\tilde{a},\tilde{b}\right)=0\right\}\triangleq Eq.(15)\end{aligned} \tag{18}$$

The degree of maximum and minimum value of $\tilde{a}$ and $\tilde{b}$ equals to α which means[9]

$$\begin{aligned}\mu_{\tilde{a}}\left(\tilde{a}_\alpha^L\right)&=\mu_{\tilde{a}}\left(\tilde{a}_\alpha^U\right)=\alpha\\ \mu_{\tilde{b}}\left(\tilde{b}_\alpha^L\right)&=\mu_{\tilde{b}}\left(\tilde{b}_\alpha^U\right)=\alpha\end{aligned} \tag{19}$$

Fuzzy mean life $\tilde{\theta}_0$ under normal stress level S_0 then will be given through Eq.(20) by invoking PSO a second time .

$$\tilde{\theta}_0 = \exp\left(\tilde{a} + \tilde{b}\ln\left(S_0\right)\right) \tag{20}$$

And fuzzy reliable distribution function is easily presented as Eq.(21) which shows the reliability of the specimen is an interval value given a certain timing point t.

$$\tilde{R}_0 = \exp\left(t / \tilde{\theta}_0\right) \tag{21}$$

IV. CASE STUDY

Data from [10] are used to illustrate the model. Four accelerated stress level are settled to assess the reliability of some product. The rated voltage of the product is 10Vol. Specific settings of the ALT are as below.

TABLE I. TEST CONDITION

Test condition and number of results	Failure time of samples(unit: hour)
S_1=15v,n_1=30,r_1=10	43，547，628，697，917，2245，2997，3001. 3123，3820
S_2=18v,n_2=30,r_2=10	50，254，342，535，818，1044，1318，1531，1671，2025
S_3=25v,n_3=20,r_3=15	56，85，145，176，423，439，498，735，763，833，890，987，1105，1391，1547
S_4=35v,n_4=20,r_4=20	11，35，42，49，51，55，65，89，98，100，113，120，123，133，133，158，171，208，219，734

As discussed above, the data obtained from the ALT are not precise. Based on fuzzy theory, the center value of fuzzy failure time is the timing point recorded. The spread width varies because the inspection interval varies under different stress levels. The exact inspection interval of this ALT is not provided so take such settings just for an example: g_{S1}=24, g_{S2}=12, g_{S3}=5, g_{S4}=1. Then the total testing time under accelerated stress level can be expressed as below:

$$\begin{aligned} &\tilde{T}_1 = (94618, 240), \tilde{T}_2 = (50088, 120) \\ &\tilde{T}_3 = (17808, 75), \tilde{T}_4 = (2707, 20) \end{aligned} \tag{22}$$

The final results are shown when given two α-cult level according to Eq.(16)-Eq.(21).

$$\begin{aligned} &\alpha = 0.3, \\ &\tilde{a}_{0.3} \in [23.1639, 23.2281] \\ &\tilde{b}_{0.3} \in [-5.1008, -5.0827] \\ &\tilde{\theta}_{0.3} \in [91024, 101190] \\ &\exp(-t/91024) \le \tilde{R}_{0.3}(t) \le \exp(-t/101190) \end{aligned} \tag{23}$$

and

$$\begin{aligned} &\alpha = 0.6, \\ &\tilde{a}_{0.6} \in [23.1774, 23.2143] \\ &\tilde{b}_{0.6} \in [-5.0969, -5.0865] \\ &\tilde{\theta}_{0.6} \in [93093, 98933] \\ &\exp(-t/93093) \le \tilde{R}_{0.6}(t) \le \exp(-t/98933) \end{aligned} \tag{24}$$

The failure time data are precise when α=1.It is just a special type of fuzzy number. The mean life of the case study in[10] is 96020h, a traditional result after common inference. This paper's result shows that

$$\tilde{\theta}_{0.3}^L < \tilde{\theta}_{0.6}^L < 96020 < \tilde{\theta}_{0.6}^U < \tilde{\theta}_{0.3}^U \tag{25}$$

The smaller the α-cut is, more imprecise the data are which lead to a bigger interval value of the mean life. Traditional results belong to the results of this paper.

V. CONCLUSIONS

A statistical model of ALT based on fuzzy theory is proposed in this paper for the first time. This model aims to solve the problem that the failure time data in ALT are not imprecise. Comparing to the traditional statistical inference which ends in the point estimation of reliability and lifetime, this approach analyzes the imprecision of data reasonably and obscures the failure data based on fuzzy theory. Combing with MLE and PSO, this approach finally provides an interval estimation. The lifetime of a product estimated by ALT cannot be so accurate that an interval assessment is more credible and reasonable.

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