Error allocation for Motion Mechanism Based on the Kinematic Accuracy Reliability

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Literature review

Mechanism kinematic accuracy reliability

It is defined as the probability of the displacement, velocity or acceleration of component meeting the specified value under the influences of various random factors.

The failure criterion

The performance function: $G(z) = \Delta - \delta < 0$

where, Δ is the allowed output error, δ is the actual output error

Reliability: $R = 1 - P\left[G(z)\right]$

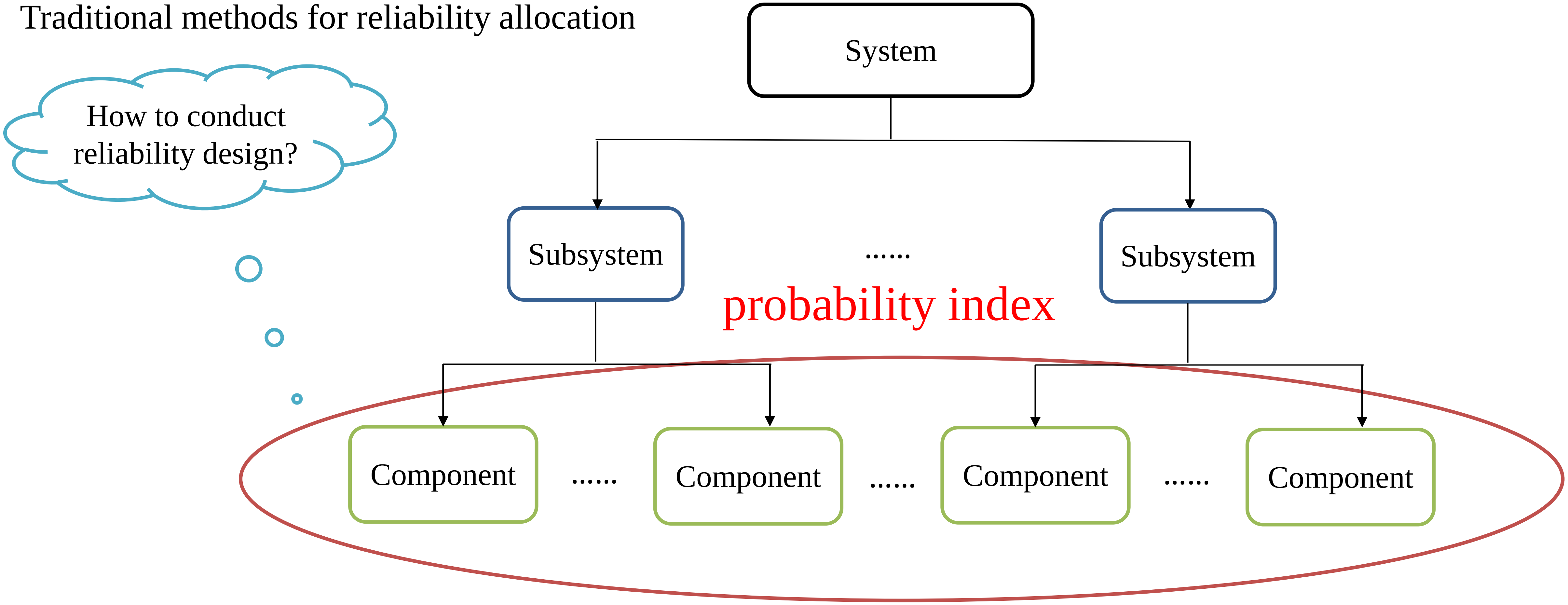
It is assumed that both the errors follow normal distribution

$$R = \phi\left(\frac{\mu_0 - \mu}{\sqrt{\sigma_0^2 + \sigma^2}}\right)$$

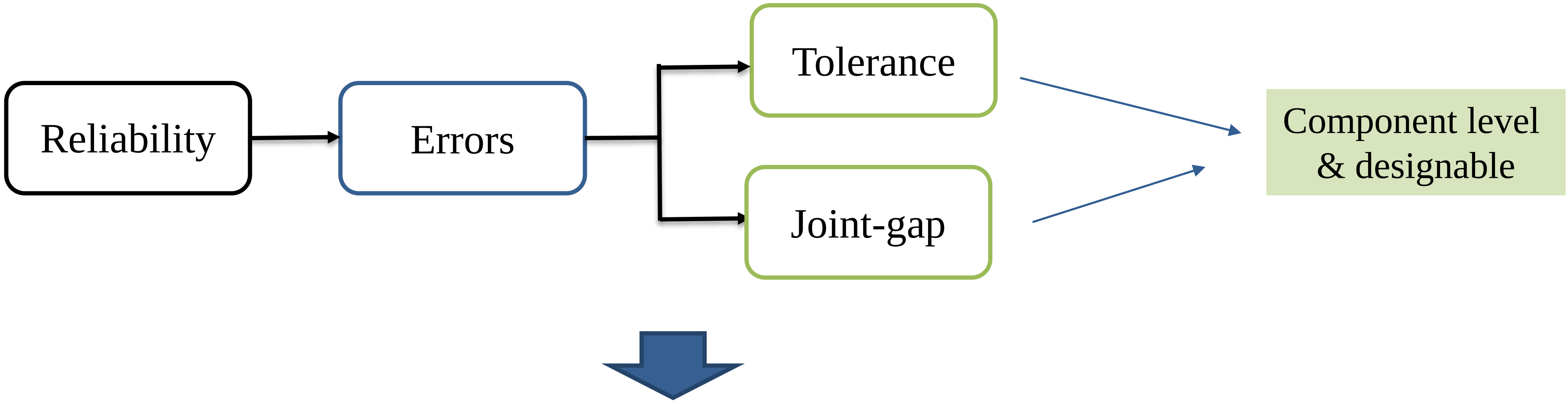
where, μ and σ^2 are the mean value and variance of the real output error, while μ_0 and σ_0^2 are the specified mean value and variance of the output error.

Literature review

Traditional methods for reliability allocation



Literature review



Our research: Error allocation Based on the Kinematic Accuracy Reliability

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1 Literature review

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3 Methodology for accuracy reliability allocation

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Reliability analysis of the kinematic accuracy for mechanism

According to:

$$R=\phi\left(\frac{\mu_0-\mu}{\sqrt{\sigma_0^2+\sigma^2}}\right)$$

where, μ_0 and σ_0^2 are the specified mean value and variance of the output error, the key is to obtain μ and σ^2 which are the mean value and variance of the actual output error.

Considered a mechanism is consist of n components linked by hinges, the output parameter is a function of the component parameters

$$Y_0=f\left(X_1,X_2,\cdots,X_n\right)$$

where, Y_0 is the output parameter in the ideal conditions, X_i (i=1,2,……,n) is the i^{th} component parameter, e.g. the bar length.

Reliability analysis of the kinematic accuracy for mechanism

In fact, the error exists in the component parameter due to the manufacturing process

$$Y = f\left(X_1 + \Delta X_1, X_2 + \Delta X_2, \cdots, X_n + \Delta X_n\right)$$

where, Y is the actual output , ΔX_i (i=1,2,……,n) is the i^{th} component parameter error.

$$\Delta Y = Y - Y_0 = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial X_i}\right)\Delta X_i$$

$$\mu = E\left(\Delta Y\right) = \sum_{i=1}^{n}\frac{\partial Y}{\partial X_i}E\left(\Delta X_i\right) = \sum_{i=1}^{n}\frac{\partial Y}{\partial X_i}\mu_i$$

$$\sigma^2 = D\left(\Delta Y\right) = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial X_i}\right)^2 D\left(\Delta X_i\right) = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial X_i}\right)^2 \sigma_i^2$$

where, μ_i and σ_i^2 are the mean value and variance of the i^{th} component parameter error, respectively.

Reliability analysis of the kinematic accuracy for mechanism

The reliability model considering joint-gaps

$$\mu = \sum_{i=1}^{n} \frac{\partial Y}{\partial X_i} \mu_i$$

$$\sigma^2 = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i} \right)^2 \left(\sigma_i^2 + \frac{\sigma_{Ri}^2 + \mu_{Ri}^2}{9} \right)$$

where, μ_{Ri} and σ_{Ri}^2 are the mean value and variance of the i^{th} joint-gap, respectively

The reliability model considering joint-gaps and wearing

$$\mu = \sum_{i=1}^{n} \frac{\partial Y}{\partial X_i} \mu_i$$

$$\sigma^2 = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i} \right)^2 \left(\sigma_i^2 + \frac{\sigma_{Ri}^2 + \sigma_v^2 t^2 + \left(\mu_{Ri} + \mu_v t \right)^2}{9} \right)$$

where, t is the working time, μ_v and σ_v^2 are the wear rate mean value and variance, respectively.

Luo, J.M. & Sun, Z.L. 2002. Research on the motion reliability design model of slider-crank mechanism. *Mechanical Science & Technology* 21(6):958-962.

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The approach for accuracy reliability allocation

Mechanism kinematic accuracy reliability

- the mean value of the component parameter error μ_i
- the variance of the component parameter error σ_i^2
- the variance of joint-gap σ_{Ri}^2
- the mean value of joint-gap μ_{Ri}

These four factors are treated as design variables, and the reliability index can be allocated to components.

we treat the reliability function as the constraint with the purpose of minimizing the cost function.

The approach for accuracy reliability allocation

The accuracy cost function:

$$G(\sigma_i) = C_i(\sigma_i - \sigma_i')^2 + D_i$$

where, C_i is a modification coefficient of machining grade, σ_i is the machining accuracy of i^{th} component, σ_i' is the lowest machining accuracy of i^{th} component, D_i is basic cost.

The tolerance cost function:

$$C(T_i) = a_i e^{-b_i T_i}$$

where, T_i is the tolerance, a and b are curve-fitting parameters that greater than 0.

According to the "3σ" principle in mechanical design, the tolerance of the component size

$$T_{ci} = 6\sigma_i$$

It is assumed that the joint-gap deviation is symmetric.

$$T_{Ri} = 2\mu_{Ri}$$

The approach for accuracy reliability allocation

The initial wear rate characteristic value (μ_v, σ_v) can be obtained if a material has been chosen. We should verify whether the wear rate meets the required reliability in the working time.

$$\mu_i = \sigma_i^2 = \mu_{Ri} = \sigma_{Ri}^2 = 0 \Rightarrow$$

$$\mu = \sum_{i=1}^{n} \frac{\partial Y}{\partial X_i} \mu_i$$

$$\sigma^2 = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i} \right)^2 \left(\sigma_i^2 + \frac{\sigma_{Ri}^2 + \sigma_v^2 t^2 + \left(\mu_{Ri} + \mu_v t \right)^2}{9} \right)$$

The optimal model for allocation

$$\min W = G(\sigma) + C(T)$$

$$s.t. \frac{\mu_0 - \mu}{\sqrt{\sigma_0^2 + \sigma^2}} \geq \phi^{-1}\left(R_0\right)$$

$$0 < \sigma_i \leq \sigma_i'$$

$$\mu_{Ri} > 0$$

The approach for accuracy reliability allocation

Choose a reasonable machining grade

Table 1 : The tolerance in different machining grade (μm)

Tolerance grade	IT5	IT6	IT7	IT8	IT9	IT10	IT11	IT12	IT13	IT14	IT15	IT16	IT17	IT18
D≤500	7k	10k	16k	25k	40k	64k	100k	160k	250k	400k	640k	1000k	1600k	2500k
500<D≤3150	7k	10k	16k	25k	40k	64k	100k	160k	250k	400k	640k	1000k	1600k	2500k

$$k = \begin{cases} 0.45\sqrt[3]{D} + 0.001D(\mu m); D \le 500mm \\ 0.004D + 2.1(\mu m); 500 < 500 \le 3150mm \end{cases}$$

where, D is the component length

$$h = \left[\frac{T_{ci}}{k}\right]$$

where, h is a maxint among these values (7, 10, 16, 25, 40, 64, 100,$\cdots$,1000, 1600, 2500) that smaller than T_{ci}/k

The final component tolerance: $T_{ci}^{'} = hk$

Wu, B. 1990. Component dimension tolerance in mechanical reliability design. *Mechanical Science and Technology* (4), 30-34.

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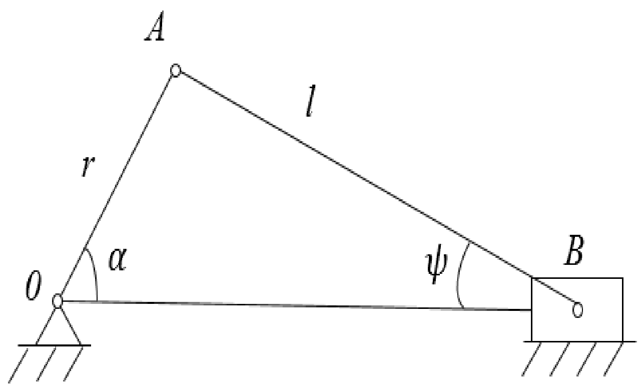
Case study

Assumption 1: The rotation angle α is an ideal value. $\Delta\alpha$=0
Assumption 2: The component dimension deviation is symmetric. μ_i=0



The Slider-crank mechanism

The length of *OA*	The length of *AB*	The rotation angle	Required reliability
r=20*cm*	*l*=40*cm*	α=90°	R_0=0.942

The error characteristic values of output displacement	The wear rate characteristic values of the material	Working time
(μ_0, σ_0)=(0.95,0.01)(*cm*)	(μ_v,σ_v)= (0.06,0.0033) (*cm/kh*)	*t*=20*kh*

Case study

The wear rate meets the required reliability ?

$$\mu=0$$
$$\sigma^2=0.2675$$
$$R=\phi\left(\frac{0.95-0}{\sqrt{0.01^2+0.2675}}\right)=0.97$$

Obviously, $R>R_0=0.942$,which indicates the meeting of error allocation.

The optimal results

Design variable	Optimal result /*cm*
σ_r	0.071
σ_l	0.139
μ_R	0.149
σ_R	0.081

$$T_r=6\sigma_r=0.426cm$$
$$T_l=6\sigma_l=0.834cm$$

$$k_r=0.45\sqrt[3]{200}+0.001\times200=2.83(\mu m)$$
$$k_l=0.45\sqrt[3]{400}+0.001\times400=3.71(\mu m)$$

According to Table 1

$$h_r=\left[\frac{T_r}{k_r}\right]=1000$$
$$h_l=\left[\frac{T_l}{k_l}\right]=1600$$

The final optimal tolerance

$$T_r'=h_rk_r=1000\times2.83\mu m=0.283cm$$
$$T_l'=h_lk_l=1600\times3.71\mu m=0.5936cm$$

Case study

The final optimal results

Design variable	Optimal result /cm
σ_r'	0.047
σ_l'	0.099
μ_R	0.149
σ_R	0.081

when a designer conducts the SCM accuracy design, the component parameters must meet the values as following component tolerance:

$$T_r' = h_r k_r = 1000 \times 2.83 \mu m = 0.283 cm$$
$$T_l' = h_l k_l = 1600 \times 3.71 \mu m = 0.5936 cm$$

Reliability allocation results:

$r = 20^{+0.1415}_{-0.1415}$ $l = 40^{+0.2968}_{-0.2968}$

The joint-gap characteristic value is (0.149,0.081)

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Conclusions

- This paper provides a new methodology for mechanism kinematic accuracy reliability allocation.
- The reliability allocation can be regarded as the error allocation by establishing the kinematic accuracy reliability function about errors considering wearing.
- The reliability index can be allocated to the component level and designed through embodying in the tolerance of component size and joint-gap.

Thank you