Nonlinear accelerated degradation analysis based on the general Wiener process

LE LIU, XIAO-YANG LI, TONG-MIN JIANG

School of Reliability and Systems Engineering
Beihang University, Beijing, China

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Contents

Literature Review

Why is Accelerated degradation testing ? – High reliability & Long lifespan VS Time constrait.

How to model the data ? – Degradation path-based or **stochastic processes** [1].

1. Clear physical explanation
2. Easy to use
3. Good properties

What is the main work of the past years ? – Wiener process with linear drift.

Is there any nonlinear data ? – Battery (relative resistance) [2], LED [3], Metal [4]

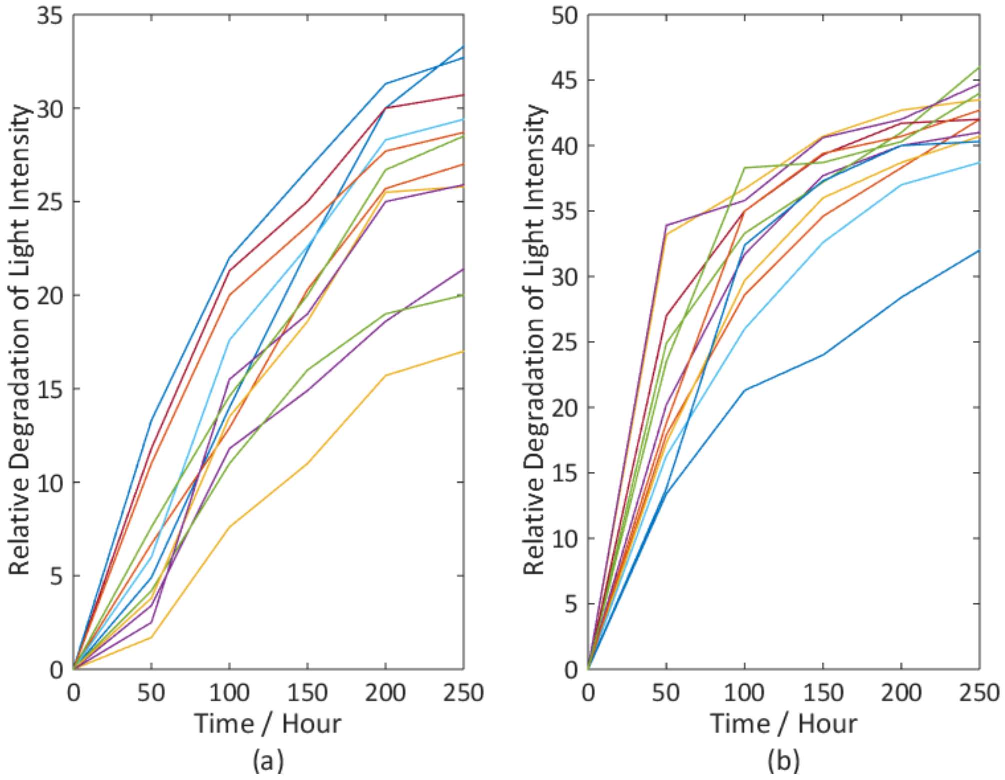
How to model the nonlinear ADT data ? – Time-scale transformation [5][6].

$$t = 1 - \exp(-\lambda r^{\gamma}) \quad \text{or} \quad t = r^{\lambda}$$

Is this model effective ? – The **scope** of this paper.

Motivating Example

24 LEDs are tested under two stress conditions: (a) 35mA and (b) 40mA [3]. The normal operating condition is 25mA.



Results in References:

In [3], the estimate of MTTF is 1346 hours through degradation-path model.

While in [6], the 95% confidence interval of the MTTF is [1848,57202] hours through time-scale transformation Wiener process model.

The differences are significant !!!

Formulas for Degradation Modeling

The time-scale transformation model [6]: $$\mathbf{M_1}: X(t) = \mu\Lambda(t;\theta) + \sigma B\big(\Lambda(t;\theta)\big) \quad (1)$$

The general Wiener process [7]: $$\mathbf{M_0}: X(t) = \mu\Lambda(t;\theta) + \sigma B\big(\tau(t;\gamma)\big) \quad (2)$$

where μ and σ are the drift (stress-related, $\mu = \eta_0 I^{\eta_1}$) and diffusion coefficients, ϑ and Υ are the two generalization parameters, $\Lambda(t;.)$ and $\tau(t;.)$ are the time-scale transformations.

Difference: the variance

$$M_0: \begin{cases} E[X(t)] = \mu\Lambda(t;\theta) \\ \& \\ Var[X(t)] = \sigma^2\tau(t;\gamma) \end{cases} \qquad M_1: \begin{cases} E[X(t)] = \mu\Lambda(t;\theta) \\ \& \\ Var[X(t)] = \sigma^2\Lambda(t;\theta) \end{cases}$$

First Passage Time Distribution

M_1: inverse Gaussian distribution $$f_1(t) = \frac{\omega}{\Lambda(t)\sqrt{2\pi\sigma^2\Lambda(t;\theta)}} \exp\left(-\frac{\left(\omega - \mu\Lambda(t;\theta)\right)^2}{2\sigma^2\Lambda(t;\theta)}\right)\frac{d\Lambda(t;\theta)}{dt} \quad (3)$$

M_0: the generalization distribution

$$p_0(t) = \frac{1}{\sqrt{2\pi\tau(t;\gamma)}}\left(\frac{\omega - \mu\Lambda(}{\sigma\tau(t;}\right. \quad \left.\frac{\Lambda(t;\theta))^2}{(t;\gamma)}\right)\frac{d\tau(t;\gamma)}{dt}$$

$$f_0(t) \cong p_0(t)/ \quad (4)$$

$$s = \tau(t;\gamma) \Rightarrow t = \tau^{-1}(s;\gamma)$$

$$\Lambda(t;\theta) = \Lambda(\tau^{-1}(s;\gamma);\theta) = \rho(s;\theta)$$

$$X(s) = \mu\rho(s;\theta) + \sigma B(s)$$

$$\kappa(s;\theta) = \mu\frac{d\rho(s;\theta)}{ds}$$

Given $\Lambda(t;\vartheta)=t^{\vartheta}$ and $\tau(t;\Upsilon)= t^{\Upsilon}$, Eq.(4) is

$$f_0(t) = \frac{\left(\omega\gamma - (\gamma - \theta)\mu t^{\theta}\right)}{t\sqrt{2\pi\sigma^2 t^{\gamma}}} \cdot \exp\left(-\frac{\left(\omega - \mu t^{\theta}\right)^2}{2\sigma^2 t^{\gamma}}\right) \quad (5)$$

If $\vartheta=\Upsilon$, then Eq.(5) is Eq.(3).

Two-stage Parameter Estimation

Definition: X_{ijk} is the kth degradation value of unit j at the stress level i, i = 1,2, … ,K; j=1,2,…,n_i; k=1,2,…,m_{ij}. t_{ijk} is the corresponding measurement time.

Let

$$\boldsymbol{X}_{ij} = \left(X_{ij1}, X_{ij2}, \ldots, X_{ijm_{ij}}\right)'$$

$$\boldsymbol{t}_{ij} = \left(\Lambda(t_{ij1};\theta), \Lambda(t_{ij2};\theta), \ldots, \Lambda\left(t_{ijm_{ij}};\theta\right)\right)'$$

$$\boldsymbol{Q}_{ij} = \begin{bmatrix} \tau(t_{ij1};\gamma) & \tau(t_{ij1};\gamma) & \cdots & \tau(t_{ij1};\gamma) \\ \tau(t_{ij1};\gamma) & \tau(t_{ij2};\gamma) & \cdots & \tau(t_{ij2};\gamma) \\ \vdots & \vdots & \ddots & \vdots \\ \tau(t_{ij1};\gamma) & \tau(t_{ij2};\gamma) & \cdots & \tau\left(t_{ijm_{ij}};\gamma\right) \end{bmatrix}$$

then,

$$\boldsymbol{X}_{ij} \sim N\left(\mu_{ij}\boldsymbol{t}_{ij}, \sigma^2\boldsymbol{Q}_{ij}\right)$$

So,

$$\hat{\mu}_{ij}|\theta,\gamma = \frac{\mathbf{X}'_{ij}\mathbf{Q}_{ij}^{-1}\mathbf{t}_{ij}}{\mathbf{t}'_{ij}\mathbf{Q}_{ij}^{-1}\mathbf{t}_{ij}} \quad (6)$$

$$\hat{\sigma}^2|\theta,\gamma = \frac{\sum_{i=1}^{K}\sum_{j=1}^{n_i}\left(\mathbf{X}_{ij} - \hat{\mu}_{ij}\mathbf{t}_{ij}\right)'\mathbf{Q}_{ij}^{-1}\left(\mathbf{X}_{ij} - \hat{\mu}_{ij}\mathbf{t}_{ij}\right)}{\sum_{i=1}^{K}\sum_{j=1}^{n_i} m_{ij}} \quad (7)$$

Two-stage Parameter Estimation

Stage 1. Parameters in degradation model ϑ, Υ, σ and μ_{ij}

- Profile likelihood function:

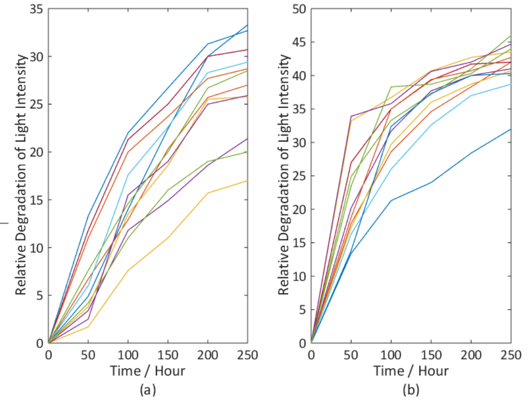
$$l(\theta,\gamma|\mathbf{X}) = -\frac{\ln(2\pi)}{2}\sum_{i=1}^{K}\sum_{j=1}^{n_i} m_{ij} - \frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{n_i}\ln\left|\hat{\sigma}^2\mathbf{Q}_{ij}\right| - \frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{n_i}\left(\mathbf{X}_{ij}-\hat{\mu}_{ij}\mathbf{t}_{ij}\right)'\hat{\sigma}^{-2}\mathbf{Q}_{ij}^{-1}\left(\mathbf{X}_{ij}-\hat{\mu}_{ij}\mathbf{t}_{ij}\right) \quad (8)$$

- ϑ and Υ can be computed by Eq.(8) through ***Fminsearch*** function in Matlab.
- Substituting ϑ and Υ to Eq.(6) and (7), σ and μ_{ij} will be given.

Stage 2. Parameters in acceleration model η_0, η_1

- MLE method:

$$\log\left(\hat{\mu}_{ij}\right) = \log\eta_0 + \eta_1 \log I_i$$



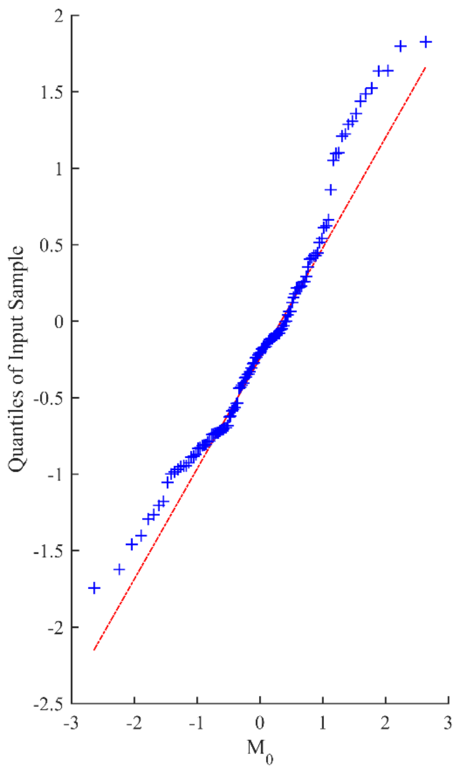
Results for the LED case

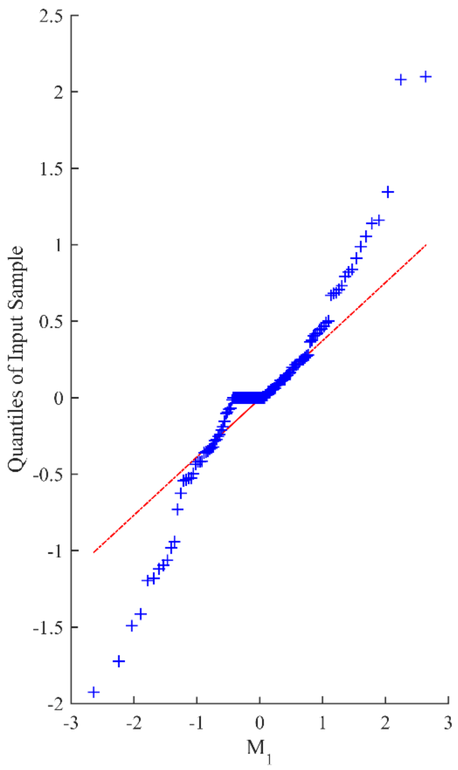
➢ Parameter Estimation

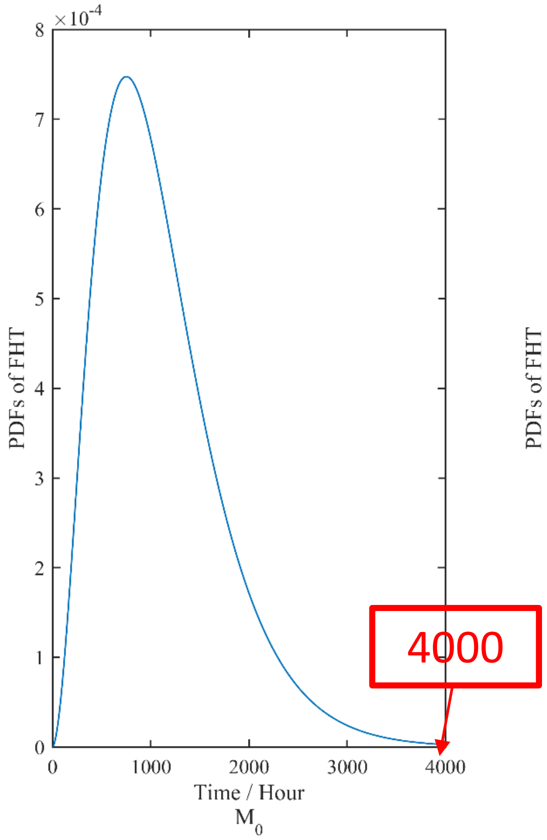
Models	Degradation Model			Acceleration model		I_{max}	AIC
	ϑ	Υ	σ^2	η_0	η_1		
M_0	0.4415	0.1172	73.7836	0.2284	0.7257	-310.39	**630.78**
M_1	0.4503		5.8400	1.804e-5	3.2968	-316.98	641.97

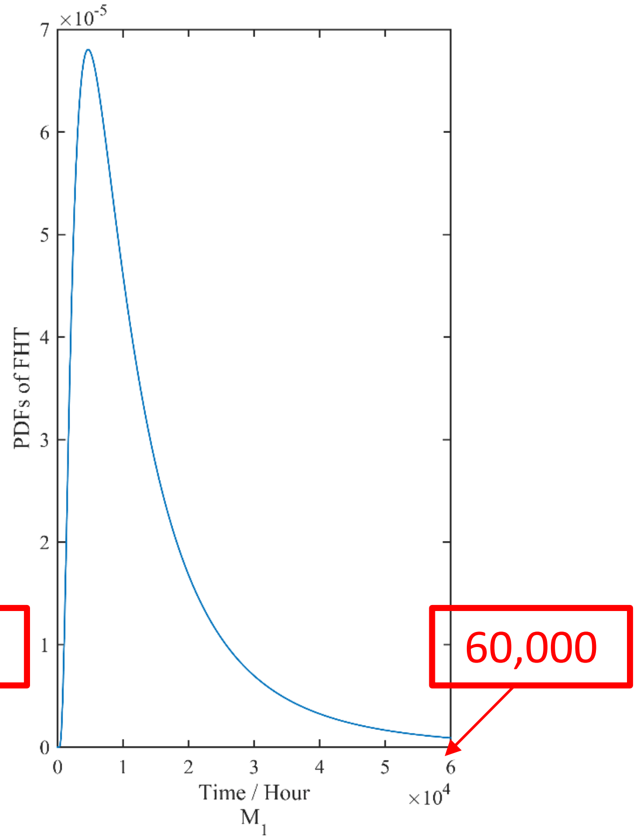
According to the AIC index, model M_0 performs better than M_1 on LED ADT data fitting. However, the differences among degradation and acceleration models are significant. The problem of mis-specification of basic degradation model is **serious**.

Model fitting and PDF of FPTs









Confidence Intervals

Models	95% CI (hours)	Comments
M_0	[232, 2622]	In accordance with the results in original paper [3], which is **1346** hours.
M_1	[1914, 53629]	Over estimated as that in [6], which is **[1848,57202]** hours

Discussions

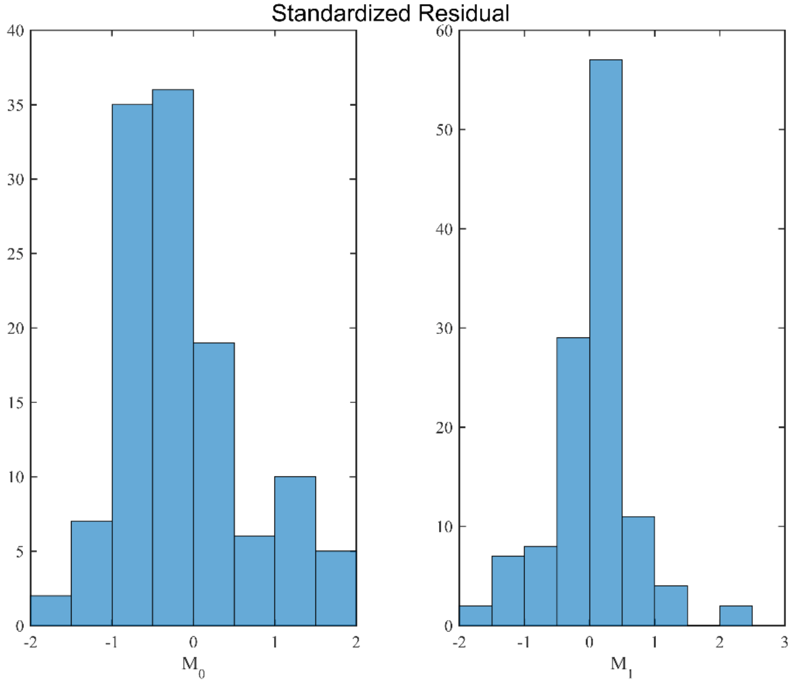
Models	Degradation Model			Acceleration model	
	ϑ	Υ	σ^2	η_0	η_1
M_0	0.4415	0.1172	73.7836	0.2284	0.7257
M_1	0.4503		5.8400	1.804e-5	3.2968

- The Variance

$$M_0: \quad Var[X] = 73.7836 \times t^{0.1172}$$

$$M_1: \quad Var[X] = 5.8400 \times t^{0.4503}$$

- It is interesting that M_0 suggests a good fitting (AIC & Q-Q plot) but has larger variance than M_1.
- The influence of variance modeling is significant on ADT analysis.



All fails to pass the hypothesis test at the 1% significance level

$Adstat_0 = 1.6775 < Adstat_1 = 3.2240$

Discussions

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M_0	0.4415	0.1172	73.7836	0.2284	0.7257
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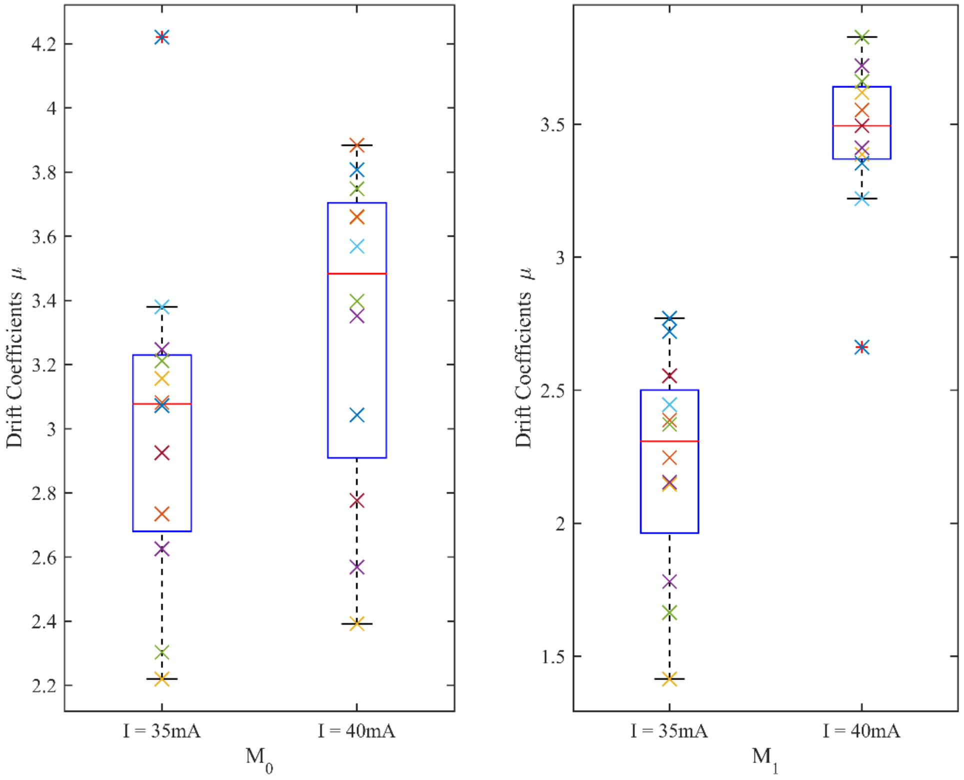
- The trap of extrapolation (acceleration model)

$$M_0: \quad \mu = 0.2284 I^{0.7257} \Longrightarrow \mu_0 = 2.3616$$

$$M_1: \quad \mu = 1.804 \times 10^{-5} I^{3.2968} \Longrightarrow \mu_0 = 0.7327$$

- M_1 give false confidence to the producers on their products with a significantly lower value of μ_0.
- This PITFALL is discussed by Meeker and Escobar (1998), named "Multiple time-scales and multiple factors affecting Degradation".



Summary

- The general Wiener process is introduced to analyze the nonlinear accelerated degradation data, which can cover the common used linear and time-scale transformation Wiener processes as its limiting cases.
- The LED case shows that this method fits better than time-scale transformation model and can provide reliable lifetime estimation results.
- When using ADT for lifetime and reliability evaluation, one should avoid the trap of extrapolation though increasing the number of stress levels and samples.

Future Work

- The consideration of unit-to-unit variation
- Nonlinear Step Stress ADT (SSADT) data
- Optimum plan design with the general Wiener process

References

[1] Ye, Z.-S. & Xie, M. 2015. Stochastic modelling and analysis of degradation for highly reliable products. Applied Stochastic Models in Business and Industry 31(1): 16-32.

[2] Thomas, E. V., Bloom, I., Christophersen, J. P. & Battaglia, V. S. 2008. Statistical methodology for predicting the life of lithium-ion cells via accelerated degradation testing. Journal of Power Sources 184(1): 312-317.

[3] Chaluvadi, V. 2008. Accelerated life testing of electronic revenue meters. Master, Clemson University.

[4] Meeker, W. Q. & Escobar, L. A. 1998. Statistical methods for reliability data, New York, John Wiley & Sons.

[5] Whitmore, G. A. & Schenkelberg, F. 1997. Modelling Accelerated Degradation Data Using Wiener Diffusion With A Time Scale Transformation. Lifetime Data Analysis 3(1): 27-45.

[6] Tang, S., Guo, X., Yu, C., Xue, H. & Zhou, Z. 2014. Accelerated Degradation Tests Modeling Based on the Nonlinear Wiener Process with Random Effects. Mathematical Problems in Engineering.

[7] Wang, X. L., Jiang, P., Guo, B. & Cheng, Z. J. 2014. Real-time Reliability Evaluation with a General Wiener Process-based Degradation Model. Quality and Reliability Engineering International 30(2): 205-220.

Acknowledgements

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Thanks and Questions!