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Utilizing Accelerated Degradation and Field Data for Life Prediction of Highly Reliable Products

Le Liu, Xiao-Yang Li,*† Tong-Min Jiang and Fu-Qiang Sun

For newly developed, highly reliable, and long-lifespan products, it is quite difficult to implement effective remaining useful life (RUL) prediction in the early usage under limited time cost. However, accelerated degradation testing (ADT) is generally used for lifetime evaluation for such products with harsher test conditions and shorter test time in the late research and development phase. Thus, in this paper, we propose a life prediction framework to integrate the information from ADT to conduct field RUL prediction for highly reliable products. Because ADT belongs to reliability testing used for inferring the population information from the selected test samples, we at first present the modified Wiener process (MWP) model. Different from traditional methods that embody both the random variability and unit-to-unit variability into the diffusion coefficient, the proposed method describes them separately in ADT analysis. Then, the MWP model from ADT is used as a prior for field RUL prediction of the target product during which the strong tracking filtering algorithm is introduced for updating the hidden state and computing the RUL prediction results when the new monitoring data are available. Because of the complexity of the MWP model, the Markov chain Monte Carlo method is provided to estimate the unknown parameters. Finally, the simulation study and the light-emitting diode application verify the effectiveness of the proposed framework that can achieve reasonable life prediction results for highly reliable products for both linear and nonlinear scenarios. Copyright © 2015 John Wiley & Sons, Ltd.

Keywords: accelerated degradation testing; reliability and life evaluation; Bayesian method; RUL prediction; Wiener process

1. Introduction

Remaining useful life (RUL) prediction is an essential part of prognostics and health management, which is also a hot research area and widely used in industrial and mechanical applications.[1,2] RUL can provide a basis for maintenance decision-making to reduce related cost and ensure the safety operation of the equipment.[3] Therefore, it is vital to guarantee the prediction accuracy, which has significant influence on the corresponding decisions. In general, the internal physical degradation process can be traced by the external change of product quality characteristics, for example, the on-state collector-emitter voltage of insulated-gate bipolar transistor[4] and the capacity of lithium-ion battery.[5] Based on which, the time to failure can be acquired by modeling the characteristic degradation index with failure threshold. For newly developed products under field usage environment, it is rather difficult to trace the degradation index in a limited time because products are born to have high reliability and long lifespan. Thus, an accurate and reasonable RUL prediction result is hard to acquire for product in usage unless enormous time cost has been put on the degradation monitoring.

In the late research and development phase, quantitative accelerated degradation testing (ADT) is widely used to verify whether the reliability and life indexes of the products satisfy the requirements before releasing them to the market.[6,7] Through more severe stress levels, performance degradation process can be accelerated and sufficient degradation data can be obtained in a limited time. Thus, ADT can effectively overcome the life evaluation problem of high reliability and long-life products.[8] Wang *et al.*[9] used ADT for life evaluation of light-emitting diode (LED)-based light bars and concluded that the failure time at normal use condition is 11 571 h. Park *et al.*[10] used organic LED as a motivation example and provided three failure time distribution inference methods from ADT data. Bae *et al.*[11] analyzed the ADT data of membrane electrode assembly and concluded the median failure time is 669.78 h. In addition, some researchers are concerned with the situation that both failure and degradation data exist in accelerated testing.[12,13] For the situation that both ADT data and field failure or degradation data exist, Wang *et al.*[14] introduced two calibration factors to modify the difference between accelerated and field data, utilizing Bayesian methods to conduct life evaluation.

Science and Technology on Reliability and Environmental Engineering Laboratory, School of Reliability and Systems Engineering, Beihang University, Beijing, China

**Correspondence to: Xiao-Yang Li, Science and Technology on Reliability and Environmental Engineering Laboratory, School of Reliability and Systems Engineering, Beihang University, Beijing, China.*
†E-mail: leexy@buaa.edu.cn

The population-based information from ADT can provide knowledge for understanding the degradation pattern of the products, which can then be used for field RUL prediction for new highly reliable products. Thus, the main interest of this paper is to propose the integration framework for the field RUL prediction of the target individual by utilizing both accelerated degradation and field data. Considering that the data from ADT are used for training the degradation model as a prior and field data can then timely update the model, less data will be needed for field RUL prediction, and reasonably accurate prediction results can be achieved in a limited time.

For most products, the analytical functions of the internal failure mechanism are hard to be obtained using physical or chemical analysis. Therefore, data-driven methods are widely used in ADT analysis[12,15,16] and life prediction area,[17,18] especially the stochastic processes based on cumulated damage theory, for example, Wiener process, Gamma process, and inverse Gaussian process.[13,19,20] With its mathematical tractability, the Wiener process is widely used as the underlying degradation model for both ADT and field data analysis. With its time-scale transformation, it can be used for both linear and nonlinear degradation analysis.[21,22]

In standard ADT analysis with a Wiener process model, it is assumed that the drift coefficient (also called degradation rate) is stress-related, and diffusion coefficient reflects both the unit-to-unit variability and random variability.[14,15,20,23] However, unit-to-unit variability should be separated from random variability because several samples are tested during ADT. Peng *et al.*[24] demonstrated this problem for degradation modeling analysis and assumed that the drift coefficient follows a normal distribution. Carey *et al.*[25] considered the influence of temperature stress on life evaluation of integrated logic family and added an error term in the acceleration model to present the unit-to-unit variability. In Section 3, the differences of reliability evaluation results due to the consideration of unit-to-unit variability are demonstrated to be significant.

For field RUL prediction, it is generally assumed that the field stress level is fixed and that the RUL prediction results can be given by substituting the field stress level to the estimated ADT model, which may be invalid in reality. In order to relax that assumption, some papers consider the field stress as stochastic distributions[26] or use calibration factors to integrate ADT and field information.[14,27] However, those methods may be less effective for products experienced in such field conditions that vary from customer to customer with their interests. From the ADT analysis, it is known that the uncertainty of field stress has influences on drift coefficient. Thus, the drift coefficient can be treated as hidden state and will be updated with new available degradation data. In this paper, the strong tracking filtering (STF) algorithm is selected for accomplishing this purpose and computing the real-time RULs because it has the strengths of real-time recursively prediction, insensitivity to shock change of the degradation process and so forth.[22,28,29] Details are referred to Zhou *et al.*[29]

The remaining sections of this paper are organized as follows. Section 2 presents the modeling methods for ADT and field data, including ADT modeling, field prediction model with RUL distributions, and the Markov chain Monte Carlo (MCMC) methods for unknown parameters. The illustration of the proposed method for both linear and nonlinear scenarios is shown in Section 3. Section 4 presents the LED case to apply to the proposed method, and discussions are given to the comparison with other two models. Section 5 concludes this paper.

2. Methodology

In this section, the integrated prediction framework based on ADT and field data is proposed. The modeling processes and the RUL distributions for both linear and nonlinear scenarios are given in detail.

In this framework (Figure 1), ADT data are used as prior information to identify the parameters of the acceleration and degradation model under accelerated conditions, in which both the unit-to unit variability and random variability are taken into consideration separately. Then, the extrapolated degradation model under normal condition is used as the initial field prediction model, and it

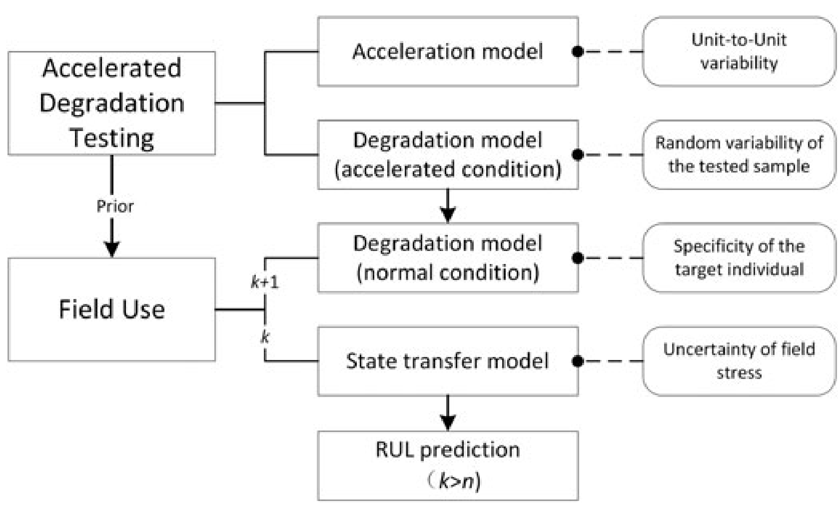

Figure 1. The life prediction framework based on accelerated degradation testing data and field data

can be updated when new monitoring data are available. Meanwhile, the uncertainty of field stress is embedded into the degradation state. Finally, the modified model is used for field RUL prediction. The proposed method can make reasonable field RUL results through using historical ADT information for high reliability and long-lifespan products. It should be mentioned that the prior information is acquired from the population of product, while real prediction in field case will be specified on the target individual with real-time monitoring data.

2.1. Acceleration and degradation models

With the great physical and statistical properties, stochastic processes are widely used for modeling the degradation process, $X(t)$, of product quality characteristic varied with time. In this paper, we select the Wiener process model with time-scale transformation.

$$X(t) = x_0 + \lambda\Lambda(t) + \sigma B(\Lambda(t)) \tag{1}$$

where x_0 is the initial degradation value. $\lambda > 0$ is the drift coefficient. $\Lambda(t)$ is the non-decreasing time transformation, and (1) is called a linear model when $\Lambda(t) = t$ and a nonlinear model when $\Lambda(t) = t^{\gamma}$. σ is the diffusion coefficient. $B(.)$ is the standard Brownian motion.

When using model (1) for ADT analysis, some assumptions are given[14,23,30]:

Assumptions:

1. Stresses are well handled to remain stable during the accelerated test.
2. Diffusion coefficient reflects both unit-to-unit variability and the variability due to operating and environmental conditions and assumes to be constant.
3. Drift coefficient governs the degradation path, which has a relationship with accelerated stresses, that is, acceleration model.

In the application of acceleration models, the log-linear relationship is a generalized form, covering Arrhenius model with temperature stress T, Eyring model with electrical stress I, and so on. For our model, that is

$$log(\lambda_i) = A + \sum_i B_i \varphi(s_i) \tag{2}$$

where λ_i is the drift coefficient under accelerated stress type i, A and B_i are constant parameters, and $\phi(s_i)$ denotes the function of accelerated stress s_i.

2.1.1. Traditional lifetime and reliability evaluation.

In standard ADT analysis, stochastic process, Eq. (1), is selected to describe the degradation process under different stress levels for each test sample. Then, the acceleration model is chosen to extrapolate the drift coefficient into normal stress s_0 by Eq. (2), that is, λ_0. Therefore, the reliability of product under normal stress can be given based on the stochastic property of the degradation process. Specifically, the first passage time (FPT) of Eq. (1) to failure threshold, D and $D' = D\text{-}x_0$, follows an inverse Gaussian distribution.[31] The probability density function (PDF) of transformed FPT and the reliability function for product are

$$f\left(t\middle|D', \sigma, \lambda_0\right) = \frac{D'}{\sqrt{2\pi\sigma^2[\Lambda(t)]^3}} exp\left\{-\frac{\left[D' - \lambda_0\Lambda(t)\right]^2}{2\sigma^2\Lambda(t)}\right\}\frac{d\Lambda(t)}{dt} \tag{3}$$

$$R\left(t\middle|D', \sigma, \lambda_0\right) = \Phi\left(\frac{D' - \lambda_0\Lambda(t)}{\sigma\sqrt{\Lambda(t)}}\right) - \exp\left(\frac{2\lambda_0 D'}{\sigma^2}\right)\Phi\left(-\frac{D' + \lambda_0\Lambda(t)}{\sigma\sqrt{\Lambda(t)}}\right) \tag{4}$$

The lifetime under the reliability level of interest can also be given by

$$t_R = \Lambda^{-1}\left[R^{-1}\left(D', \sigma, \lambda_0\right)|_{R=R_d}\right] \tag{5}$$

where R_d is the defined reliability level, t_R is the corresponding lifetime, and Λ^{-1} is the inverse function for time transformation.

2.1.2. New insight for the consideration of unit-to-unit variability.

Because several samples are put into ADT and each of them has their own degradation pattern due to the manufacturing and/or environmental condition, the unit-to-unit variability should be separated from the diffusion coefficient in Eq. (1). Therefore, we modify assumptions (2) and (3) as follows.

Assumptions:

4. Diffusion coefficient reflects the random property of the tested samples and assumes to be constant.
5. Drift coefficient is a random variable representing the unit-to-unit variability due to manufacturing and/or environmental condition.

Then, a random noise term is added into the acceleration model, that is, Eq. (2), to present the unit-to-unit variability

$$\log(\lambda_i) = A + \sum_i B_i \varphi(s_i) + \eta \tag{6}$$

where $\eta \sim N(0, \sigma_1^2)$. Thus, the drift coefficient follows lognormal distribution rather than simple normal distribution, for example, $\lambda_0 \sim LogN\left(A + \sum_i B_i \varphi(s_{i0}), \sigma_1^2\right)$.

The new marginal PDF of FPT and the reliability function relied on λ_0 are

$$f^{new}\left(t \middle| D', \sigma\right) = \int f\left(t \middle| D', \sigma, \lambda_0\right) f(\lambda_0) d\lambda_0 \tag{7}$$

$$R^{new}\left(t \middle| D', \sigma\right) = 1 - \int_0^{\Lambda(t)} f^{new}\left(u \middle| D', \sigma\right) du \tag{8}$$

Meanwhile, the reliable lifetime can also be given by

$$t_R^{new} = \Lambda^{-1}\left[R^{new-1}\left(D', \sigma\right)|_{R=R_d}\right] \tag{9}$$

Hence, the integration of Eqs (1) and (6) is the modified Wiener process (MWP) for ADT analysis, which separates the random variability and unit-to-unit variability into degradation and acceleration models. The identified MWP with ADT data can then be used as a prior prediction model for field RUL prediction.

2.2. *On-line life prediction for field use*

Assumptions:

6. No new failure mechanism is introduced when products are used in field operating conditions.
7. Products follow the same degradation pattern; thus, the random variability in normal condition is the same with that in accelerated test.

In this paper, we consider the situation that products experienced a harsh or gentle but not destructive environment in field use, which satisfies assumption (6). In terms of assumption (7), the consistency of product property means that they follow the same pattern in gradual degradation. Therefore, the degradation process in field use is in accordance with Eq. (1).

Without loss of generality, the uncertainty of field stress is presented on the time-varying degradation state λ, which is stress-related (Eqs (2) and (6)). Thus, the state transfer model at time k can be simplified as follows

$$\lambda_k = \lambda_{k-1} + \delta_{k-1} \tag{10}$$

where $\delta_{k\text{-}1}$ denotes the influence of field stress level on degradation state at time $k-1$, which is assumed to be *s*-independent and *s*-normal distributions with mean 0 and standard deviation σ_2. The initial value of degradation state is given from ADT by taking field stress level s_0 into Eq. (6).

The field degradation history till time k is described by $X_{0:k}$, $X_{0:k} = [X(t_0), X(t_1), \ldots, X(t_k)]'$. According to Eq. (1) and assumption (7), the field measurement model is given according to the MWP.

$$x(t_k) = x(t_{k-1}) + \lambda_{k-1}\Delta t_k + \sigma\varepsilon_k \tag{11}$$

where $\Delta t_k = \Lambda(t_k)\text{-}\ \Lambda(t_{k\text{-}1})$, where $\varepsilon_k \sim N(0, \Delta t_k)$.

Hence, Eqs (10) and (11) constitute the basic prediction model for product in field use, and the drift coefficient, λ, can be treated as hidden variable and estimated from the field monitor history. When new degradation data are available, the state and measurement equations can be updated for RUL prediction. With the failure threshold D, RUL at time k, L_k, can be defined as the FPT, which satisfies

$$L_k = \inf\{l_k > 0 | x(t_k + l_k) \geq D, x(t_k) < D\} \tag{12}$$

In practical field use, the degradation process may experience a shock change rather than gradual deterioration. Hence, the STF algorithm is introduced to update the hidden state. Details about the algorithm are given in the following:

Algorithm 1: hidden state estimation

Step 1: Setting initial value

$$\hat{\lambda}_0 = E(\lambda_0) = e^{\left(A + \sum_i B_i \varphi(s_{i0}) + \sigma_1^2/2\right)}, P_{0|0} = Var(\lambda_0) = \left(e^{\sigma_1^2} - 1\right) e^{\left(2\left(A + \sum_i B_i \varphi(s_{i0})\right) + \sigma_1^2\right)}, \alpha, \rho$$

Step 2: Calculating fading factor $r(t_k)$ from the orthogonality principle

$$
\begin{aligned}
B(t_k) &= V_0(t_k) - \sigma_2^2 \Delta t_k{}^2 - \alpha\sigma^2 \Delta t_k \\
V_0(t_k) &= \begin{cases} v^2(t_k) & k = 1 \\ \dfrac{\rho V_0(t_{k-1}) + v^2(t_k)}{1+\rho} & k > 1 \end{cases} \\
v(t_k) &= x(t_k) - x(t_{k-1}) - \widehat{\lambda}_{k-1}\Delta t_k \\
C(t_k) &= P_{k-1|k-1}\Delta t_k{}^2 \\
r(t_k) &= \max\{1, B(t_k)/C(t_k)\}
\end{aligned}
$$

Step 3: Estimating the hidden state by

$$
\begin{aligned}
P_{k|k-1} &= r(t_k)P_{k-1|k-1} + \sigma_2^2 \\
Q_k &= \Delta t_k{}^2 P_{k|k-1} + \sigma^2 \Delta t_k \\
\widehat{\lambda}_k &= \widehat{\lambda}_{k-1} + P_{k|k-1}\Delta t_k Q_k^{-1} v(t_k)
\end{aligned} \tag{13}
$$

Step 4: Updating the variance of the hidden state by

$$
P_{k|k} = P_{k|k-1} - P_{k|k-1}\Delta t_k{}^2 Q_k^{-1} P_{k|k-1} \tag{14}
$$

In Algorithm 1, the softening factor α and the forgetting factor ρ can be selected heuristically where we set $\rho = 0.95$.[29] Based on Eqs (13) and (14), the PDF of hidden variable λ_k replied on degradation history $X_{0:k}$ is

$$
f(\lambda_k|X_{0:k}) = \frac{1}{\sqrt{2\pi P_{k|k}}} \exp\left(-\frac{\left(\lambda_k - \widehat{\lambda}_k\right)^2}{2P_{k|k}}\right) \tag{15}
$$

where $\widehat{\lambda}_k$ *and* $P_{k|k}$ denote the mean and variance of variable λ at time k.

2.2.1. Remaining useful life distribution for linear scenario. We first consider linear scenario as the simple case when $\Lambda(t) = t$. The FPT of (11) follows an inverse Gaussian distribution as Eq. (3). At time k, the RUL of product relied on the degradation history $X_{0:k}$ and state variable λ_k is

$$
f(l_k|X_{0:k}, \lambda_k) = \frac{D - x(t_k)}{\sqrt{2\pi (l_k)^3 \sigma^2}} \cdot \exp\left(-\frac{(D - x(t_k) - \lambda_k l_k)^2}{2\sigma^2 l_k}\right) \tag{16}
$$

where state variable λ_k is not observable and should be estimated recursively based on historical degradation data till current time k.

Considered the uncertainty of field stress, the PDF of RUL is computed by Eqs (15) and (16) using the total probability rule.[28]

$$
f(l_k|X_{0:k}) = \int f(l_k|X_{0:k}, \lambda_k) f(\lambda_k|X_{0:k}) d\lambda_k \tag{17}
$$

That is

$$
f(l_k|X_{0:k}) = \frac{D - x(t_k)}{\sqrt{2\pi (l_k)^3 \left(\sigma^2 + P_{k|k} l_k\right)}} \cdot \exp\left(-\frac{\left(D - x(t_k) - \widehat{\lambda}_k l_k\right)^2}{2\left(\sigma^2 + P_{k|k} l_k\right) l_k}\right). \tag{18}
$$

2.2.2. Remaining useful life distribution for nonlinear scenario. With the complexity of nonlinear property, the analytical form of the FPT for Eq. (11) is hard to be given. However, under some mild assumptions,[22,32] the result is

$$
f(l_k|X_{0:k}) = \frac{\gamma (t_k + l_k)^{\gamma - 1}(D - x(t_k))}{\Delta\Lambda(t_k + l_k)\sqrt{2\pi U_k}} \cdot \exp\left(-\frac{\left(D - x(t_k) - \widehat{\lambda}_k \Delta\Lambda(t_k + l_k)\right)^2}{2U_k}\right). \tag{19}
$$

where $\Delta\Lambda(t_k + l_k) = (t_k + l_k)^\gamma - t_k^\gamma$, $U_k = \Delta\Lambda(t_k + l_k)^2 P_{k|k} + \sigma^2 \Delta\Lambda(t_k + l_k)$.

Obviously, Eq. (18) is the special case of Eq. (19) when $\gamma = 1$.

2.3. Parameter estimation

This section addresses the estimates of unknown parameters, which is $\Theta = [A, B_i, \sigma, \sigma_1, \sigma_2, \gamma]$. From aforementioned knowledge, $[A, B_i, \sigma, \sigma_1, \gamma]$ are related to ADT data and describe the population information of products, while σ_2 for the target product in field use. Thus, the parameter set Θ can be divided into two parts: $\Theta_1 = [A, B_i, \sigma, \sigma_1, \gamma]$ and $\Theta_2 = [\sigma_2]$, and be estimated separately.

2.3.1. Estimation of Θ_1. In terms of the ADT data, (1) and (2) can easily lead to

$$\Delta x_{ijk} \sim N\left(\lambda_{ij}\Delta t_{ijk}, \sigma^2 \Delta t_{ijk}\right) \tag{20}$$

$$log\left(\lambda_{ij}\right) \sim N\left(A + \sum_i B_i \varphi(s_i), \sigma_1^2\right) \tag{21}$$

where x_{ijk} is the performance degradation value at accelerated stress level i of sample j at monitor point k, $i = 1,2,\ldots,K$; $j = 1,2,\ldots, n_i$; $k = 1,2,\ldots, m_j$, and $\Delta x_{ijk} = x_{ijk} - x_{ij(k-1)}$, $\Delta t_{ijk} = \Lambda(t_{ijk}) - \Lambda(t_{ij(k-1)})$. Thus, the joint likelihood function parameter Θ_1 based on ADT data is

$$\begin{aligned} L(X|\Theta_1) &= p\left(\Delta x_{ijk}\middle|\lambda_{ij}, \sigma, \gamma\right) p\left(\lambda_{ij}|A, B_i, \sigma_1\right) \\ &= \prod_i \prod_j \left[\prod_k f\left(\Delta x_{ijk}\middle|\lambda_{ij}, \sigma, \gamma\right) f\left(\lambda_{ij}|A, B_i, \sigma_1\right)\right] \end{aligned} \tag{22}$$

The posterior distribution is

$$\pi(\Theta_1|X) = L(X|\Theta_1)\pi_0(\Theta_1) \tag{23}$$

where $\pi_0(\Theta_1) = \pi_0(A, B_i, \sigma, \sigma_1, \gamma)$ is the joint prior distribution for unknown parameters.

As the complex expressions for unknown parameters Θ_1 from (20) to (23), it is quite difficult to acquire the analytical results. However, the form can be treated as a hierarchical model from the perspective of Bayesian analysis. Thus, the MCMC and advanced sampling method can be used to obtain the posterior PDFs of the parameters. The mean values of the computational results are treated as the parameters' estimation. Then, reliability evaluation for ADT and the prior information for field life prediction can be obtained by (8), (10), and (11).

In this paper, Gibbs sampling method is selected with WinBUGS to evaluate the parameters of interest.[33] We assume that the priors of the unknown parameters are non-informative and independent. Specifically,

$$\begin{gathered} A \sim N\left(\mu_A, \varepsilon_A^2\right),\ B_i \sim N\left(\mu_{B_i}, \varepsilon_{B_i}^2\right) \\ \sigma \sim IGa(a, b),\ \sigma_1 \sim IGa(a_1, b_1) \end{gathered}$$

and

$$\gamma \sim N\left(\mu_\gamma, \varepsilon_\gamma^2\right)$$

2.3.2. Estimation of Θ_2. The parameter Θ_2, that is, σ_2, represents the influence of field environmental stress on the degradation state of the target product. Therefore, it can be estimated through the degradation history, that is, $X_{0:k}$. According to Eqs (10) and (11), we have

$$\lambda_k \sim N\left(\lambda_{k-1}, \sigma_2^2\right) \tag{24}$$

$$\Delta x(t_k) \sim N\left(\lambda_{k-1}\Delta t_k, \sigma^2 \Delta t_k\right) \tag{25}$$

It is quite similar to the form of that in the estimation of Θ_1, which formulates a hierarchical model and can be timely updated when new measurement are available. Therefore, we also use MCMC method to estimate σ_2. A similar estimation procedure can be found in Wang *et al.*[22]

3. Simulation results

In this section, we use simulation tests to illustrate the proposed method for both linear and nonlinear scenarios, which is to tell the using of ADT data for field RUL prediction. By analyzing the prediction results, some comments are concluded for the proposed method. For the purpose of the comparison of traditional and new ADT evaluation results, as given in Eqs (4) and (8), we call the situation that considers the influence of unit-to-unit variability as case 1, and the other one is case 2.

3.1. *Simulation of linear scenario*

The Wiener process model in Eq. (1) is used to generate the linear degradation data with $\Lambda(t) = t$, and the parameter values are given in Table I. The simulation data for both step stress ADT and field use with single stress are shown in Figure 2(a) and (b).

3.1.1. Parameter estimation for accelerated degradation testing analysis. At first, the unknown parameters in ADT, that is, $\Theta_1 = [A, B, \sigma, \sigma_1]$, are estimated by following the procedure shown in Section 2.3.1. The Gelman–Rubin index is chosen to check the convergence of Markov chains, which is the degree of approximating 1.[34] We set the total iteration to 200 000 times, and the first 50 000 results is discarded as the burn-in period, after that the two chains become stable. The estimated posterior PDFs for ADT parameters are given in Figure 3(a), while the Gelman–Rubin indexes are given in Figure 3(b). It is obvious that all the sampling chains for parameters are converged. Table II shows the estimation results with 95% confidence intervals and the corresponding relative errors calculated by

$$Error = |(Par_{est} - Par_{real})/Par_{real}| \times 100 \tag{26}$$

As shown in Table II, the proposed parameter estimation method in this paper is reasonably accurate. Thus, it can provide reliable prior information for field RUL prediction with the estimated degradation model. Table III shows the parameter estimation results for

Table I. Simulation parameters for step stress accelerated degradation testing

Conditions		Contents					Values
Accelerated states		Stress (T/°C)					60, 80, 100
		Monitor points at each stress level					50, 30, 20
		Monitor interval (hours)					5
		Number of samples					6
		Acceleration model					$\varphi(s) = 1/(273.15 + T)$
Normal state		Stress (T/°C)					25
		Monitor times					17
		Monitor interval (hours)					1000
		Number of samples					2
Model parameters	y_0	D	A	B	σ	σ_1	σ_2
	$N(0,0.25)$	25	12	−5500	0.01	0.5	0.0001

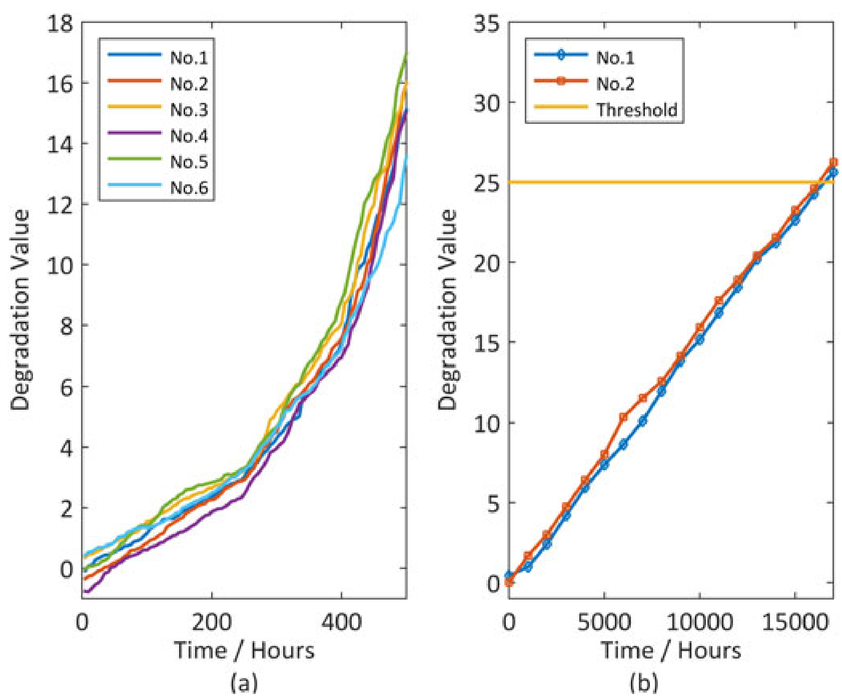

Figure 2. (a) The simulated six step stress accelerated degradation testing samples and (b) two field samples for linear scenario

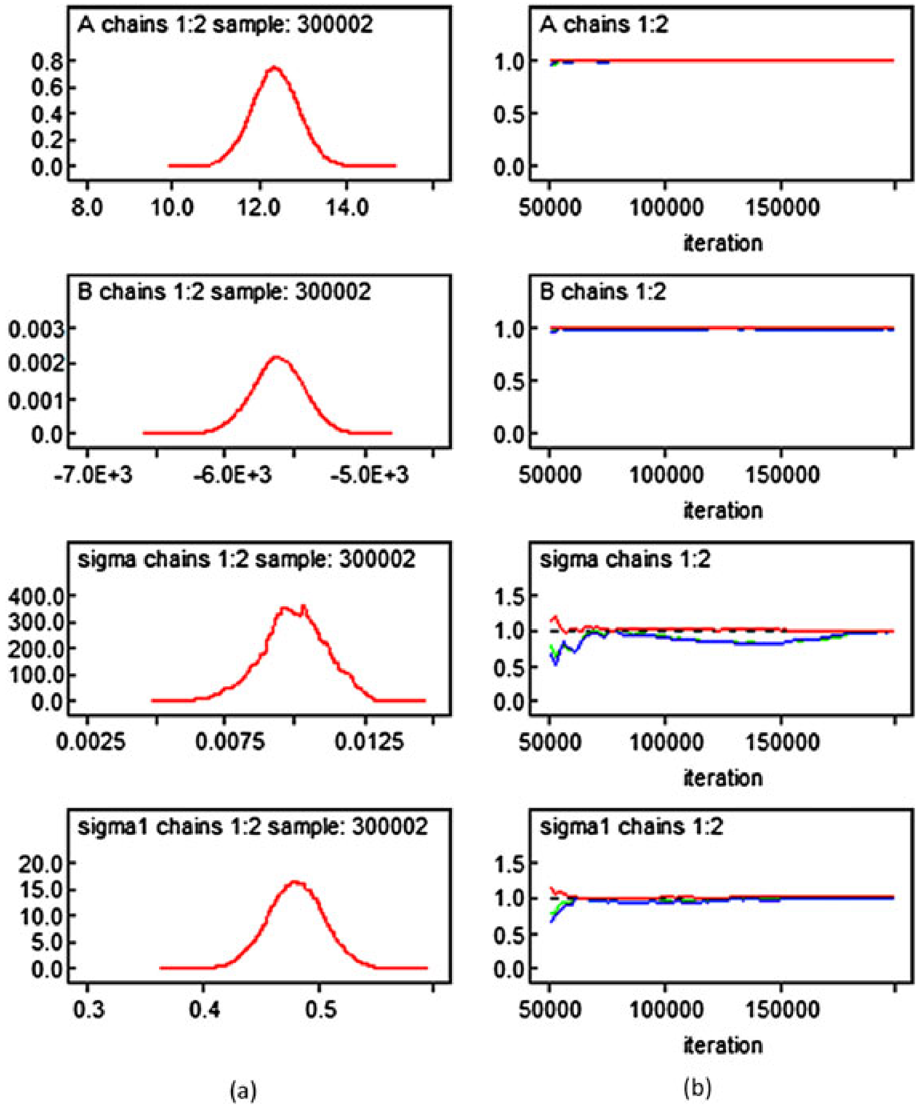

Figure 3. (a) Posterior probability density functions of step stress accelerated degradation testing parameters and (b) convergence check

Table II. Case 1: the parameter estimation results and related errors

Parameters	mean (Par_{est})	std	2.5%	97.5%	*Error* (%)
A	12.3617	0.5340	11.32	13.41	3.01
B	−5621.9	186.8	−5990.0	−5256.0	2.22
σ	0.0097	0.0011	0.007768	0.01223	3.32
σ_1	20.4808	0.0243	0.4319	0.5274	3.84

std, stands for standard deviation.

case 2. It is interesting to conclude that the parameters in acceleration models (2) and (6) are relatively close to true values in two cases because their related errors are lower than 3%. However, as we can see from σ, the value in case 2 is significantly far from the true one. The reason may be that it contains not only the effect of random property of degradation process but also the unit-to-unit variability, which means that such differences are not distinguishable as that in case 1.

In terms of the evaluation results, the reliability curves for the two cases are given in Figure 4, which offer an intuitive understanding of the difference between two cases. For the same set of ADT data, the reliability and lifetime evaluation results vary from one to another. Specifically, case 1 embodies the variability of degradation process to both drift and diffusion coefficients, while case 2 only to diffusion coefficient. Thus, the reliability curve for case 1 is more stable than case 2 and close to actual values. In practical applications, the confidence interval is also of interest to present the uncertainty of parameter, which will be computed by using bootstrap method. The 95% confidence interval for case 1 is given in Figure 4, which can capture the real reliability curve.

Table III. Case 2: the parameter estimation results and related errors

Parameters	mean (Par_{est})	std	2.5%	97.5%	*Error* (%)
A	12.16	0.6763	11.03	13.47	1.34
B	−5549.4	239.9	−6016.0	−5150.0	0.90
σ	0.0211	0.0013	0.01866	0.0237	110.86

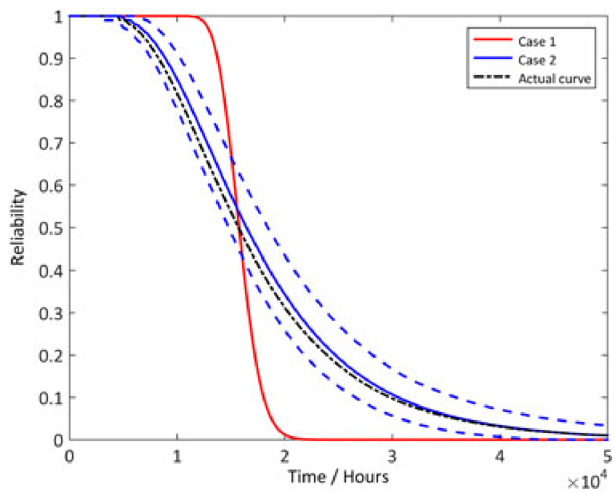

Figure 4. The reliability curves in two cases and the compared true curve for linear scenario

If the producers care about the lifetime when reliability level equals to 0.9, the values are 8900(h) ([7600, 10 300](h)) and 13 754(h) according to Eqs (5) and (9), while the true value is 8400(h). Therefore, special attention may be needed for the unit-to-unit variability in ADT analysis and reliability evaluation.

3.1.2. Life prediction for field use. In actual field use, the RUL for the target product is that of interest that influences the maintenance policy and decision-making. For illustration purpose, two degradation paths are simulated with 17 monitor degradation values, and both of paths exceed the failure threshold as shown in Figure 2(b). Thus, it can be treated as the total life cycle degradation data to verify the correctness of the prediction results.

The parameter estimation results from ADT in Section 3.1.1 are taken as prior information of models (10) and (11). The initial values can be set accordingly, that is, $\widehat{\lambda}_0$ =0.001697, $P_{0|0}$ = 7.4906e-07, while the coefficient parameter is σ = 0.0097. At each monitor point, the hidden variable can be updated with time when the new degradation value is available by Algorithm 1 and Eq. (18). Thus, the corresponding PDFs of RULs can be computed. Figure 5 shows the estimated PDFs of the RULs before failure with the actual values. Intuitively, the estimated results become more accurate with the accumulated degradation history.

We also calculate the relative error of the predicted degradation paths with the true ones (Figure 6), which presents that the predicted paths are quite close to true paths except for the first monitor point, which relies only on the initial value. Thus, the information from ADT analysis can be used for field life prediction as prior information and linear degradation path modeling.

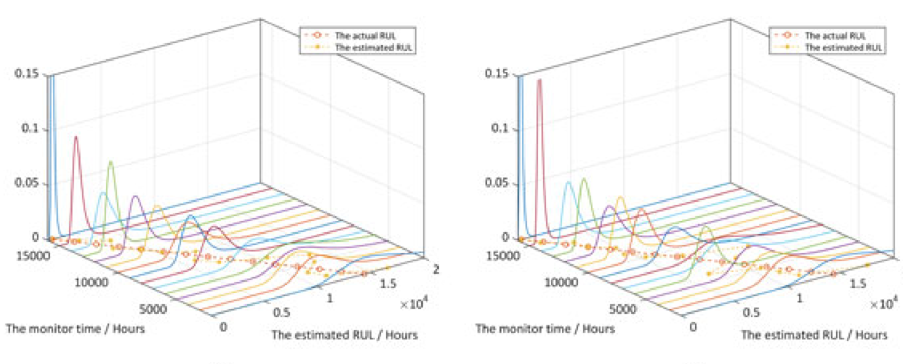

Figure 5. The estimated probability density functions of remaining useful life for sample 1 (a) and sample 2 (b) from the second monitor point to that before failure

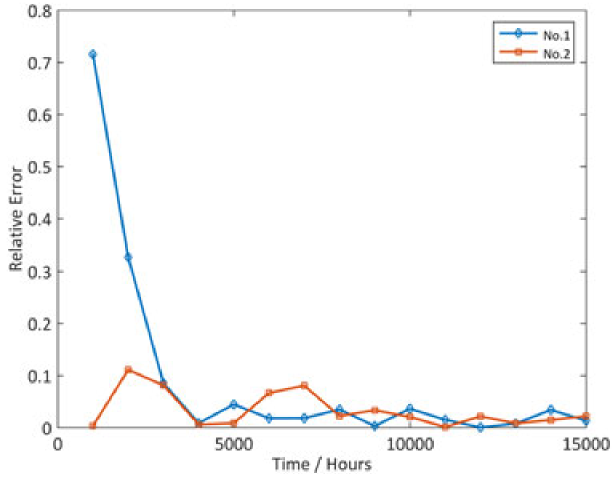

Figure 6. Relative error for the predicted degradation paths with the true paths for linear scenario

Table IV. Simulation parameters for constant stress accelerated degradation testing

Conditions		Contents					Values
Accelerated states		Stress (T/°C)					50, 65, 75
		Monitor points at each stress level					100
		Monitor interval (hours)					5
		Number of samples					12 (4 samples in each stress level)
		Acceleration model					$\varphi(s) = 1/(273.15 + T)$
Nonlinear		γ					1.5
Normal state		Stress (T/°C)					25
		The number of monitoring					13
		Monitor interval (hours)					300
		Number of samples					2
Model parameters	y_0	D	A	B	σ	σ_1	σ_2
	$N(0,0.25)$	25	11	−6000	0.01	0.5	0.0001

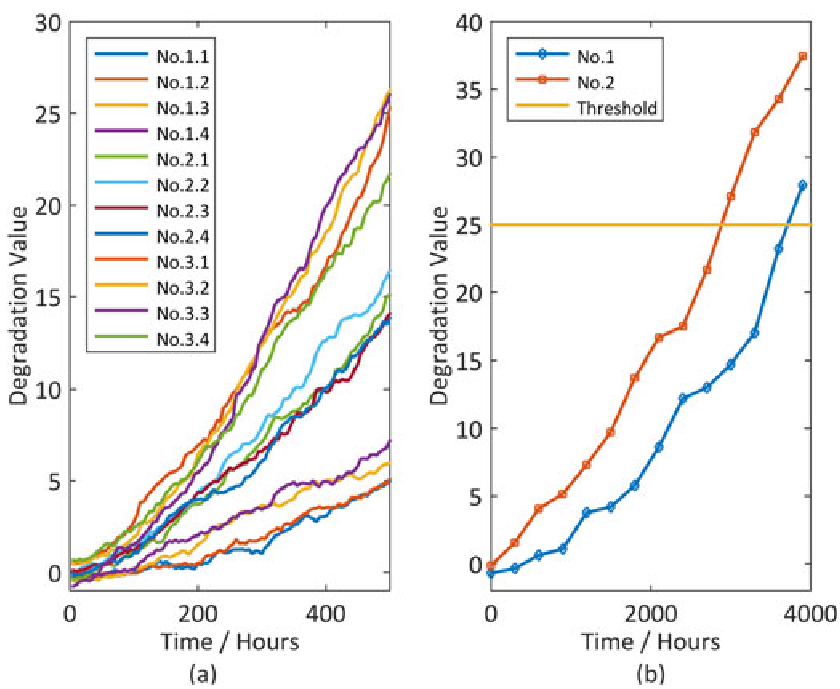

Figure 7. (a) The simulated 12 constant stress accelerated degradation testing samples and (b) two field samples for nonlinear scenario

3.2. Simulation of nonlinear scenario

For the simulation procedure of nonlinear degradation, it is quite similar to linear scenario. The time-scale transformation is chosen to be $\Lambda(t) = t^{\gamma}$[21, 22, 32], and the parameter values are given in Table IV. The simulation data for both constant stress ADT (CSADT) and field use with single stress are shown in Figure 7(a) and (b).

3.2.1. Parameter estimation for accelerated degradation testing analysis.

For the two cases, the parameters in ADT model, that is, $\Theta_1 = [A, B, \sigma, \sigma_1, \gamma]$, are estimated by MCMC method following the procedure in Section 2.3.1. Two hundred thousand iterations are simulated for the purpose of convergence check of the sampling Markov chains, while the first 50 000 is abandoned as burn-in period (Figure 8). Tables V and VI present the estimation results with 95% confidence intervals for two cases. The nonlinear parameter, that is, γ, in case 1 is more accurate than that in case 2. Compared with the linear scenario, the diffusion in case 2 is also larger than that in case 1 but not that much significant, while the parameter σ_1 has a lower accuracy maybe because of small sample.

For the reliability and lifetime evaluation, Figure 9 shows the results for two cases and the true value. Similar with the linear scenario, case 1 is much close to true one and stable than case 2, which means that the consideration of unit-to-unit variability is

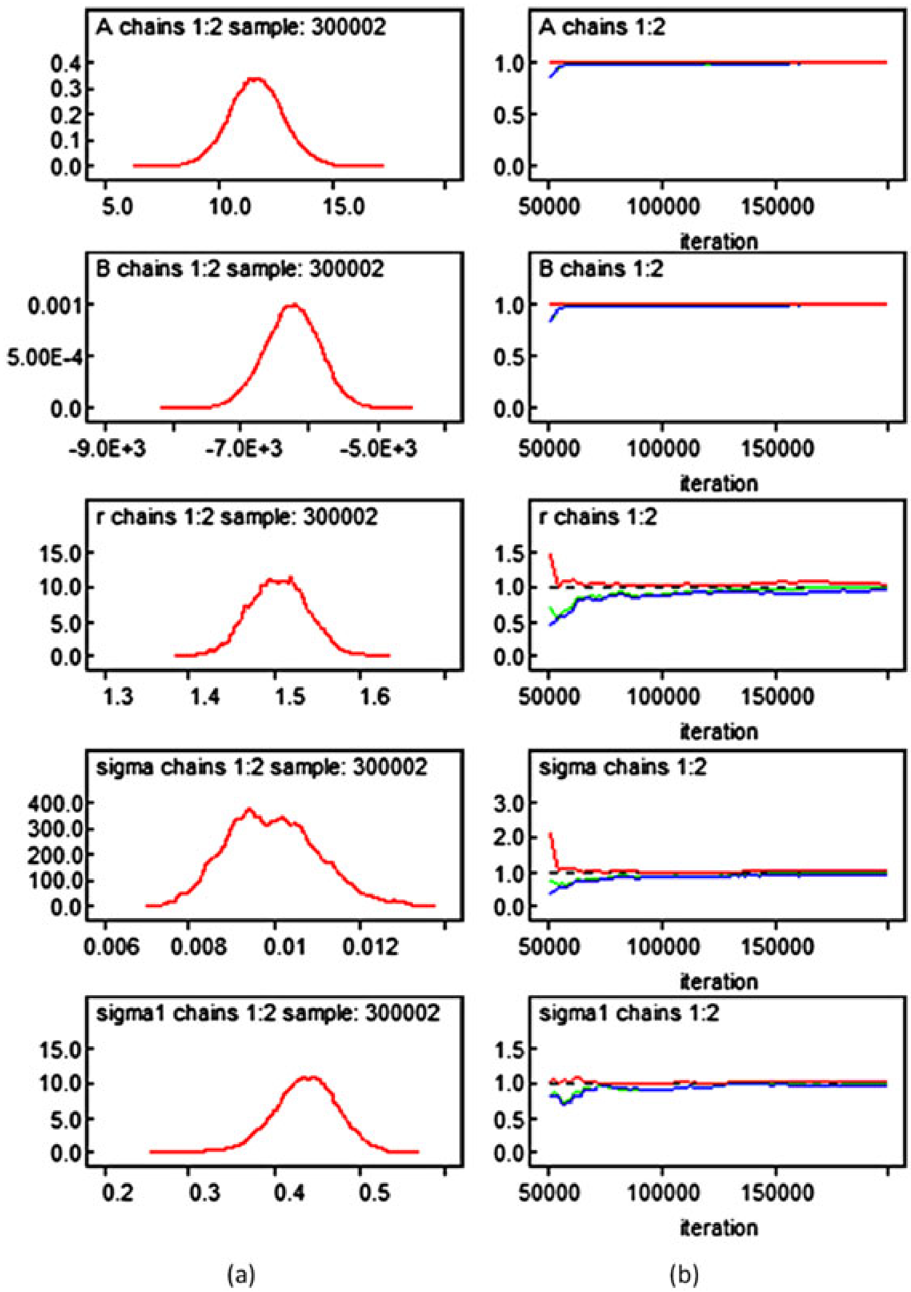

Figure 8. (a) Posterior probability density functions of constant stress accelerated degradation testing parameters and (b) convergence check

Table V. Case 1: the parameter estimation results and related errors					
Parameters	mean (Par_{est})	std	2.5%	97.5%	*Error* (%)
A	11.7207	1.164	9.506	14.07	6.55
B	−6264.6	396.4	−7062.0	−5511.0	4.41
γ	1.5089	0.02694	1.455	1.562	0.60
σ	0.0098	8.347E-4	0.008264	0.01159	1.72
σ_1	0.4320	0.03881	0.3539	0.5067	13.61

Table VI. Case 2: the parameter estimation results and related errors					
Parameters	mean (Par_{est})	std	2.5%	97.5%	*Error* (%)
A	10.5522	1.15	8.795	13.04	4.07
B	−5904.4	388.2	−6713.0	−5305.0	1.59
γ	1.5413	0.02971	1.482	1.6	2.76
σ	0.0104	9.77E-4	0.008629	0.01246	3.94

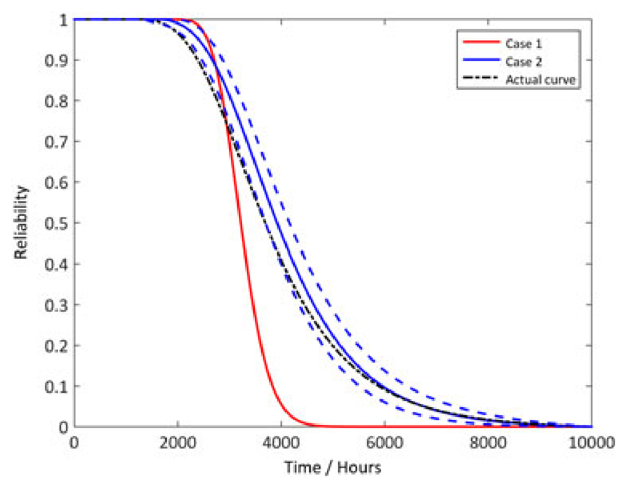

Figure 9. The reliability curves in two cases and the compared true curve for nonlinear scenario

needed for ADT analysis. The 95% confidence interval for case 1 is also given in Figure 9, which is slightly deviated from the true curve but performs better than case 2.

If the reliability of 0.9 is that of interest, the values are 2670(h) ([2470, 2870](h)) and 2701(h), respectively, comparing with the true value 2400(h).

3.2.2. Life prediction for field use. Then, we calculate the RULs for the two target products in Figure 7(b), which have 13 monitor points, and the two paths exceed the failure threshold. Thus, the total life cycle degradation data can verify the correctness of the prediction results.

The parameter estimation results from ADT in Section 3.2.1 are taken as prior information of models (10) and (11), which are $\widehat{\lambda}_0$ =1.0128e-04, $P_{0|0}$ = 2.1039e-09, while the coefficient parameter σ = 0.0098 and nonlinear parameter γ = 1.5089. At each monitor point, the corresponding PDFs of RULs can be computed by Algorithm 1 and Eq. (19). Figure 10 shows the estimated results with the actual ones before failure. Meanwhile, with the accumulating of degradation data, the estimated RULs are more accurate especially when it is quite close to failure.

Figure 11 presents the relative error of the predicted paths for two field products, which experience a gradual decrease when the influence of initial value is negligible. Thus, the information from ADT analysis can also be used for field life prediction as prior knowledge and nonlinear degradation path modeling.

4. Discussion

In this section, a real engineering application is used for applying the proposed method, which has been demonstrated to be effective for ADT modeling and field RUL prediction of both linear and nonlinear scenarios in Section 3, and discussions are given to the

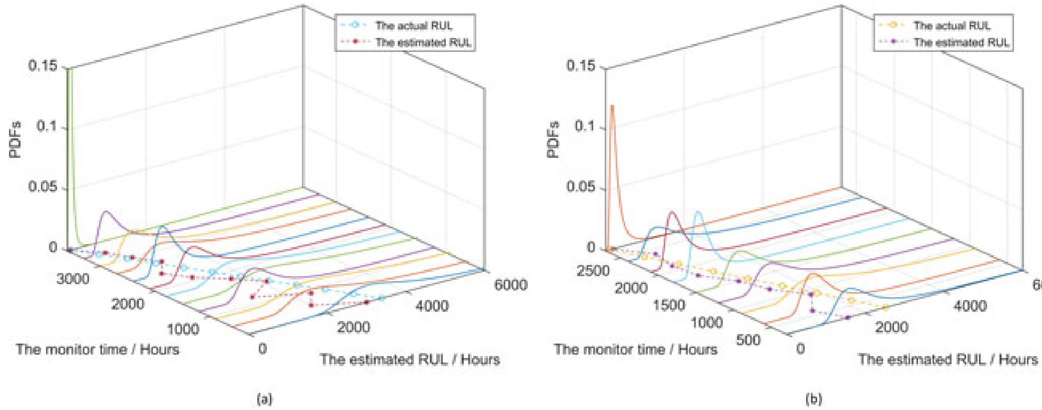

Figure 10. The estimated probability density functions of remaining useful lives for sample 1 (a) and sample 2 (b) before failure

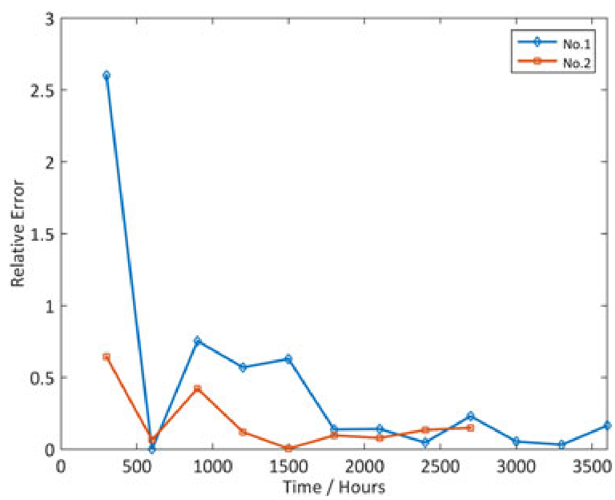

Figure 11. Relative error for the predicted degradation paths with the true paths for nonlinear scenario

comparison of the reliability evaluation results from the proposed method with the other two models. The CSADT data for LED in Liao *et al.*[26] is selected, which has four accelerated stress levels (Table VII). The normal stress is 40 °C in temperature and 10 mA in current. One simulated LED degradation data in field use are given in Table VIII. The LED fails when its light intensity drops 50% of the initial value.

The original performance data (the light intensity) are transferred into relative degradation by

$$x_i = \frac{x_0 - x_i}{x_0} \tag{27}$$

where x_i and $x_0 = 150$ are the i^{th} and initial degradation data (Figure 12 and Table VIII).

It is obvious that the LEDs experienced a nonlinear degradation path. Therefore, the $\Lambda(t) = t^{\gamma}$ is chosen for ADT modeling and field life prediction. In the simulation study, the unit-to-unit variability is argued in ADT analysis. We further verify it from the viewpoint of model selection. Deviance information criterion (DIC) is generally used for Bayesian hierarchical model selection, the one with the smaller value fits the best.[35]

$$DIC = \overline{D(\theta)} + p_D \tag{28}$$

Table VII. Constant stress acceleration degradation testing for light-emitting diode

Conditions	Contents	Values
Accelerated states	Stress 1 and 2 (T/°C, I/mA)	140 40, 140 35, 165 40, 165 35
	Monitor point (hours)	50, 100, 150, 200, 250
	Number of samples	20 (5 light-emitting diodes in each stress level)
	Acceleration model	$\varphi(s_1) = 1/(273.15 + T)$, $\phi(s_2) = \log(I)$

Table VIII. Light-emitting diode degradation value for field use

Contents	Monitor value					
Monitor point (hours)	0	1000	2000	3000	4000	5000
Light intensity (lumen/m^2)	150	126.74	112.77	97.20	90.36	84.74
Relative degradation	0	0.1551	0.2482	0.3520	0.3976	0.4369

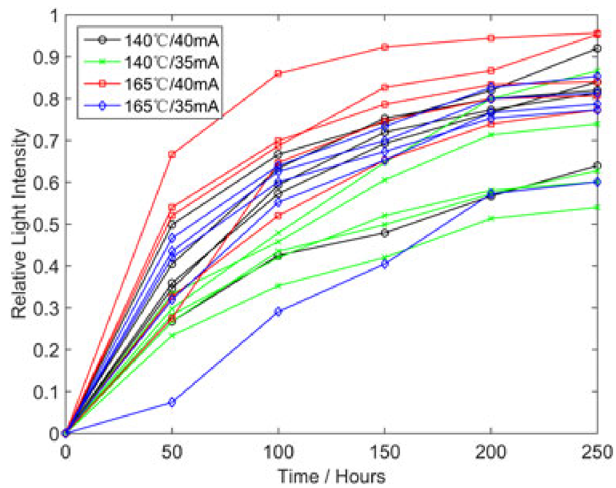

Figure 12. Constant stress accelerated degradation testing data for light-emitting diodes

where p_D is the number of effective parameters and $\overline{D(\theta)} = E^{\theta}[D(\theta)]$ is the posterior mean deviance relied on the unknown parameter θ and $D(\theta) = -2\log(p(x|\theta))$ obtained from Eq. (22). Table IX presents the estimation results, which indicate case 1 is the best model for ADT analysis with a lower DIC value that demonstrates the presence of unit-to-unit variability.

We also compute the reliability results of our model (case 1) with that in Liao *et al.*[26] and Wang *et al.*[14] for comparison, shown in Figure 13. Note that Liao's model mainly considered the variability of future stress and nominated the diffusion coefficient to be a function of accelerated stress (constant in this paper, see assumptions (2) and (4)). The variability of future stress definitely affects the drift coefficient of Eq. (1) in a dynamic behavior; however, it cannot be ensured as stochastic. The model in Eq. (10) is more general for capturing this variability mainly in field RUL prediction phase. For Wang's model, two calibration factors are introduced to modify the difference between ADT and field data on drift and diffusion coefficients, which is effective in the case when field data are sufficient. However, for the LED case, only five data in Table VIII are available for the estimates of two calibration factors. Thus, the problem of small samples causes the results of reliability curve to significantly deviate from the proposed method (Figure 13). Meanwhile, Wang's model fails to separate the unit-to-unit variability from the random variability, which demonstrates the superiority of the proposed method.

The results from Table IX are then used as prior information for field RUL prediction. Figure 14 shows the estimated PDFs of RULs at each monitor point, which can be further used for maintenance decision-making.

Table IX. The parameter estimation results for light-emitting diode constant stress accelerated degradation testing data in two cases

	Parameters	Mean (Par_{est})	std	2.5%	97.5%	Deviance information criterion
Case 1	A	−3.71	3.20	−9.477	2.57	−301
	B_1	−716.4	698.4	−2155.0	577.0	
	B_2	0.64	0.72	−0.7157	2.028	
	γ	0.42	0.025	0.3689	0.4707	
	σ	0.034	0.0073	0.01925	0.04764	
	σ_1	0.16	0.11	0.003685	0.3691	
Case 2	A	−3.79	2.422	−8.438	1.129	−280
	B_1	−943.8	567.5	−2126.0	180.8	
	B_2	0.9418	0.5649	−0.1727	1.983	
	γ	0.42	0.025	0.3737	0.4726	
	σ	0.040	0.0053	0.03107	0.05169	

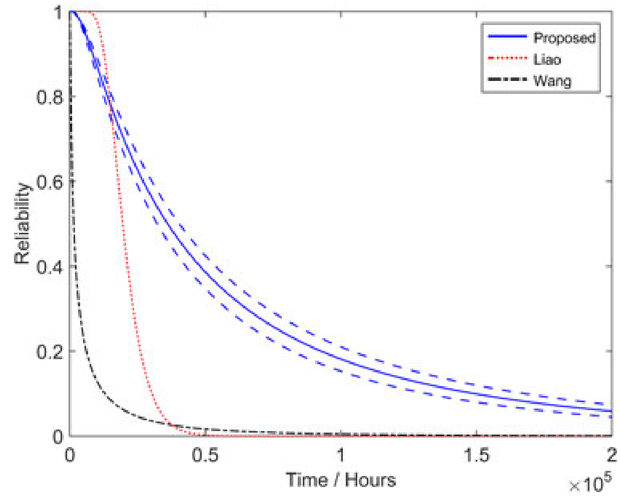

Figure 13. The reliability curves for the proposed, Liao, and Wang models

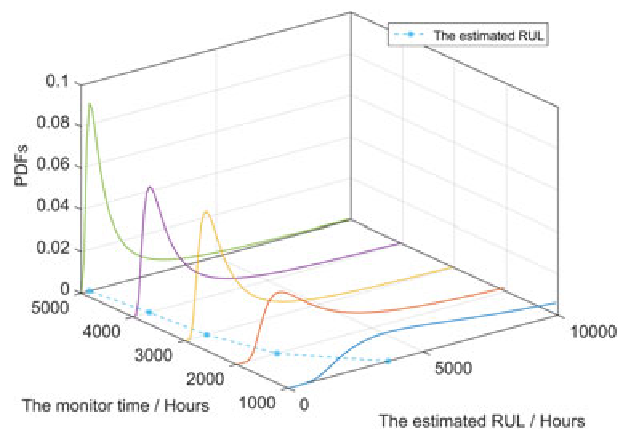

Figure 14. The probability density functions of remaining useful life (RULs) for light-emitting diode life prediction

5. Conclusions

In this paper, a field RUL prediction framework based on ADT data is proposed based on MWP model for highly reliable products. In this framework, the ADT data from late research and development phase is used for MWP model identification at accelerated conditions with the consideration of unit-to-unit variability. The identified MWP model at normal condition is used as the initial degradation model for field RUL prediction, which will be updated when new measurement are available through STF algorithm. Unknown parameters are estimated based on the proposed MCMC method. The results in Tables II and V have shown its effectiveness through Bayesian hierarchical analysis. From the simulation study and the real LED application, it is known that the unit-to-unit variability has a significant effect on life prediction results (Figures 4, 9, and 13). The results also demonstrate the effectiveness of the proposed framework on field RUL prediction for both linear and nonlinear scenarios.

The paper aims for field RUL prediction by utilizing accelerated degradation information with field monitoring data. However, such integration is still restricted to some assumptions. Further research may be given to other data-driven methods according to the property of degradation path, like Gamma or inverse Gaussian processes.[17,36] Meanwhile, model uncertainty may also need to be considered in both ADT analysis and field RUL prediction as each model has its pros and cons for life prediction. We hope to report our finding on this issue in the future.

Acknowledgements

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Authors' biographies

Le Liu is a PhD candidate at the School of Reliability and Systems Engineering, Beihang University. He received his Bachelor's degree in Quality and Reliability Engineering from Beihang University in 2012. His research interests include reliability modeling, accelerated testing, and uncertainty analysis.

Xiao-Yang Li is an associate professor at the School of Reliability and Systems Engineering, Beihang University. She received her PhD degree in Aerospace Systems Engineering from Beihang University in 2007. Her research interests include accelerated testing, prediction technology, and imprecise probability modeling.

Tong-Min Jiang is an eminent professor at the School of Reliability and Systems Engineering, Beihang University. He received his Master's degree in Reliability Engineering from Beihang University in 1990. His research interests include reliability testing and environment engineering of product.

Fu-Qiang Sun is a lecturer at the School of Reliability and Systems Engineering, Beihang University, and a visiting scholar at the Department of Industrial Engineering, University of Arkansas. He received his PhD degree in Systems Engineering from Beihang University in 2012. His research interests include accelerated degradation/life testing, durability analysis and verification, prognostics, and multivariate statistical analysis.